

Students' Mathematical Generalization in Solving Numeracy Problems

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ABSTRAK

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ABSTRACT

Generalisasi adalah komponen penting dari aktivitas matematika dan telah mendapatkan perhatian yang meningkat dalam matematika sekolah di semua tingkatan. Penelitian ini bertujuan untuk menganalisis kemampuan generalisasi matematis siswa SMP pada materi pola bilangan. Penelitian ini merupakan penelitian deskriptif dengan pendekatan kualitatif. Subyek penelitian ini adalah siswa Sekolah Menengah Pertama yang berjumlah 28 siswa pada semester genap. Siswa tersebut terdiri dari 12 siswa laki-laki dan 16 siswa perempuan. Metode pengumpulan data yang digunakan adalah metode tes. Analisis data dalam penelitian ini dilakukan melalui tahap reduksi, penyajian, dan penarikan kesimpulan terhadap data yang diperoleh. Berdasarkan hasil analisis data menunjukkan kemampuan generalisasi matematis siswa SMP pada pola bilangan diperoleh: (1) 8 siswa (28%) memiliki kemampuan generalisasi matematis tinggi, (2) 15 siswa (54%) memiliki kemampuan sedang, dan (3) 5 siswa (18%) mempunyai kemampuan rendah. Temuan ini menunjukkan bahwa sebagian besar siswa mempunyai kemampuan generalisasi matematis yang baik, namun ada pula yang masih memerlukan pendampingan lebih lanjut dalam meningkatkan kemampuannya.

Generalization is an important component of mathematical activity and has received increasing attention in school mathematics at all levels. This research aims to analyze junior high school students' mathematical generalization abilities in number pattern material. This research is descriptive research with a qualitative approach. The subjects of this research were 28 junior high school students in the even semester. The students consisted of 12 male students and 16 female students. The data collection method used is the test method. Data analysis in this research was carried out through the stages of reduction, presentation and drawing conclusions on the data obtained. Based on the results of data analysis, it shows that the mathematical generalization ability of junior high school students in number patterns was obtained: (1) 8 students (28%) had high mathematical generalization ability, (2) 15 students (54%) had medium ability, and (3) 5 students (18%) have low ability. These findings show that the majority of students have good mathematical generalization abilities, but there are also those who still need further assistance in improving their abilities.

1. INTRODUCTION

Generalization is one of the emphasized skills in mathematics learning. Generalization the way of communicating and thinking, is the main foundation for constructing mathematical knowledge and is at the center of core activities in the field. The importance of generalization is also reflected in the curriculum standards and the research conducted in mathematics education (Dinarti & Qomariyah, 2022; Iswara et al., 2022; Saefurohman et al., 2021). Generalization skills in mathematics learning are considered a key component of the mathematics learning process because they are important across various topics. (Chua & Hoyles, 2014; Dani et al., 2017). Generalizing is central to mathematical activities that enable the construction of new knowledge. Furthermore, generalization is also crucial for understanding different mathematical concepts, and generalization skills are one of the most important skills in mathematics learning (Callejo & Zapatera, 2017; Karabulut & Özmen, 2018). Understanding patterns is an important step in building generalization skills. We can identify hidden relationships and rules behind data or mathematical situations by understanding patterns. Patterns are thus a critical foundation for forming more complex and

in-depth generalizations (Aristawati et al., 2018; Mata-Pereira & da Ponte, 2017; Suhirman et al., 2020). Researchers often use pattern materials in both numerical and geometric forms as tasks to test generalization skills (Joanna, 2017; Mulenga & Marbán, 2020). Patterns are key concepts that are important in understanding mathematical knowledge and concepts. Several mathematical concepts can be developed using patterns in different contexts. Thus, the generalization ability of an individual can be revealed through his or her response to problems involving patterns.

The generalization of number patterns is identifying similarities among the terms in a pattern to find a general formula for the n-th term. The importance of learning the generalization of number patterns and object configurations has been recognized at the elementary through secondary levels (Anderson et al., 2016; Pittig et al., 2018). The meaningfulness of learning number pattern generalization for students is as follows: (1) it promotes inductive abilities, problem-solving, numeracy, relational reasoning, and initiates modeling; (3) there is a conceptual relationship between number pattern generalization and mathematical structures; and (4) generalization is at the core of mathematical activities and is important for constructing new knowledge and reasoning (Hayuningrat & Rosnawati, 2022; Jeannotte & Kieran, 2017). Therefore, it is very important to provide students with instruction in generalizing number patterns at different levels of education.

TIMSS 2019 results show that Indonesian students have a low level of mathematical generalization ability, especially in cognitive thinking (reasoning), with a percentage of only 17% (Ambussaidi & Yang, 2019; Hadi & Novaliyosi, 2019). Learning should focus on developing students' ability to infer from specific statements to improve this ability (Rosnaeni, 2021; Sapa'at, 2020). Therefore, this study analyze the mathematical generalization ability of junior high school students based on certain indicators. Hopefully, the results of this study will provide an overview of the level of generalization ability that can be categorized as high, medium, or low. Thus, the learning process can further involve students in the development of their generalization ability, and the results of this study can be a reference for research and teachers' consideration in choosing appropriate methods to achieve more effective learning goals.

This study is innovative in revealing students' generalization process in high, moderate, and low ability categories as they solve problems by analyzing Mason's indicators in detail. The results of this study have important benefits in providing students with an understanding of the steps involved in generalizing patterns and a theory that teachers can use to teach patterns to students who may experience errors in their generalization (Gal et al., 2020; Miller, 2018; Saefurohman et al., 2021). Given the important role of teachers in the learning process at school, this research can help improve the quality of learning and also assist teachers in providing more effective teaching related to the concepts of pattern generalization.

2. METHOD

This research uses a descriptive qualitative research design intending to describe the analysis of the mathematical generalization ability of junior high school students in solving numerical problems. Data on mathematical generalization ability were obtained through tests administered to students. The resulting data are descriptive in the form of writing that reflects the actual situation (Anggito & Setiawan, 2018). The definition of qualitative analysis is used as a basis in this study, where the focus is on understanding the development of research subjects, such as behavior, perceptions, motivations, and actions, which are described in the form of words and language, with attention to the natural context and using natural methods (Budiyono, 2017; Umrati, 2020). The researcher compiled four questions covering each generalization indicator. Two experts in mathematics learning then validated the test instrument prepared. Table 1 is used as a reference for the indicators in the test to measure mathematical generalization ability by Mason.

Indicators	Description
Perception of	Perception or pattern identification process
Generality	1. Able to recognize a rule or pattern
	2. Knowing that the problem presented can be solved using a rule or pattern.
Expression of	Process of using pattern identification results to determine the next data
Generality	3. Use the results of pattern identification to determine the next structure/data/image/sequence
	4. Able to describe a rule or pattern both numerically and verbally
Symbolic Expression of	Process of symbolically formulating generality
Generality	5. Able to generate a general rule or pattern
	6. Able to formulate generality symbolically

Table 1. Indicators of Mathematical Generalization Test Instrument

Indicators	Description
Manipulation of	Process of using generalization results to solve problems
Generality	7. Using generalization results to solve problems
	8. Apply rules or patterns that have been found in various problems

The mathematical generalization test used in this study consisted of four questions and is presented in Figure 1.

. Dalam suatu perlombaan baris-berbaris suatu regu merencanakan untuk membuat pola barisan dengan pimpinan grup berada di tengah seperti ditunjukan pada gambar berikut:



Berdasarkan informasi tersebut pilihlah Benar atau Salah pernyataan berikut serta tuliskan alasan yang sesuai dengan data!

- a. Setiap bertambah pola, maka penambahan anggota pada formasi barisan bertambah dengan tetap.
- b. Pada formasi pola ke-5 maka jumlah anggota seluruhnya adalah 17 orang.
- c. Jika terdapat 30 orang, maka agar membentuk barisan yang sesuai dengan pola di atas, maka 1 orang harusnya tidak diikutkan kedalam barisan.
- d. Jika pola tersebut terbentuk dari pola ke-1 sampai pola ke-10 jumlah anggota yang terlibat adalah 200.

Figure 1. Mathematical Generalization Test Question

This study involved 28 students from class VIII MTs Negeri 1 Surakarta in the even semester of the 2022/2023 school year. The students consisted of 12 male students and 16 female students. The research subjects were chosen because they had learned number pattern material and had previously faced problems with numeracy literacy style. Data analysis in this study was carried out through the phases of reduction, presentation, and concluding the data obtained. In each category of mathematical generalization ability, two students were randomly selected to represent each category. Table 2 represented High, moderate, and low categorization is based on.

Table 2. Student Mathematical Generalization Category

Category	Criteria
High	$score > \overline{X} + 0.5s$
Moderate	$\overline{X} - 0.5s \le score \le \overline{X} + 0.5s$
Low	$score < \overline{X} - 0,5s$

Description: *X* = Student's average score; *s* = Deviation standard

The research subjects were coded as follows: students with high generalization ability using the SKT code, students with moderate generalization ability using the SKS code, and students with low generalization ability using the SKR code.

3. RESULT AND DISCUSSION

Result

In this research, the mathematical generalization test questions were given to MTs Negeri 1 Surakarta students in the classroom so that researchers could directly see the process of working. The results of the analysis of mathematical generalization test questions from 28 students are presented in Table 3. Based on Table 3 show the results of the data analysis, two students from each category were selected. Six student works are used to analyze their mathematical generalization ability in solving numeracy problems. The results of the analysis of the answers to the test questions covering four indicators of mathematical generalization ability, which are Perception of generality, Expression of generality, Symbolic expression of generality, and Manipulation of generality with high, moderate, and low categories can be seen in Table 4.

Table 3. Mathematical Generalization Test Analysis Results

0	Score Interval	Students	Percentage
High	score > 90	8	28%
Moderate	$73.5 \le score \le 90$	15	54%
Low	score < 73.5	5	18%

Table 4. Analysis of Mathematical Generalization Ability

Indicator	Category	Percentage	Average
	High	91%	
Perception of Generality	Moderate	82%	79%
	Low	66%	
	High	100%	
Expression of Generality	Moderate	75%	77%
	Low	56%	
	High	100%	
Symbolic Expression of Generality	Moderate	80%	78%
	Low	55%	
	High	99%	
Manipulation of Generality	Moderate	93%	81%
	Low	50%	

Students with High Mathematical Generalization Ability (SKT)

SKT-01 could recognize that the problem in the question item was a pattern. At the *Perception of generality* phase, shown in Figure 2.

1.	4.	-ISE WAK	krn	Set	iap ł	paris	on f	d q	zamb.	ar	bar to	ambah
	-	A Cem	pat)	Sett	ap poli	2		-				
	B.	Salah,	Po krn	Po	la ke	1.	5 k	.e.2	= 9	ke s	; 13	ke. 1= 17
		ke	:5 :	21				4				
T	C.	Benar	krn	Pola	ke:6	= 29	5 Pola	he	7 =	29		
	°Q	Sarah	krn	Por	a he b	: 3	3 heg	= 37	, l	c 10 =	- 41	

Figure 2. SKT-01 Answer

Base on Figure 2 SKT-01 showed this by stating that four were added in each row. SKT-01 was able to see that each pattern always increases steadily. The pattern generalization strategy used by SKT-01 was counting. SKT-01 counted the number of squares and then realized that the number of squares corresponded to the order of the pattern. Response shows that SKT-01 knew the next cadence pattern could be obtained using a certain rule. SKT-01 could describe the rule or pattern numerically at the *Expression of* the generalization phase. As shown in Figure 2, SKT-01's answer to question b shows this. SKT-01 found that the number of members in the fifth team line pattern was 21. SKT-01 identified the pattern in the problem and then used the identification result to determine the next term. As seen in SKT-01's answer, he could numerically describe the rules of the problem. For example, in question item b, SKT-01 can find the number of members in the fifth team's line pattern. At the *Symbolic expression of the generalization* phase, SKT-01 was able to produce a rule in general. SKT-01 showed this with the answer to the question point c. SKT-01 wrote that the pattern of the lineup of the seventh team is 29. So SKT-01 was able to find the right reason for the problem. Furthermore, at the Manipulation of the generality phase, SKT-01 solved the problem by using the results of his generalization. SKT-01 applied the rule or pattern he found to answer the question in item d. SKT-01 showed the correct answer that the total number of members involved was 230. So, it can be seen from SKT-01's answer, which describes them one by one based on the results of his identification in the previous initial phase. Next, the work of SKT-02 in completing the mathematical generalization test is presented in Figure 3.

6.)	salah, seluruhnya ada 20 anggota dan 1 pemimpin (21)
	karena setiap pola ke-n ditambah 4 n
c.)	benar, n-7 · 28. Jika ingin- Maka anggota harw ditambah 1 orang as
	bisa menjadi 30 anggota dan ditambah 1 pemimpin agar menjadi 30 oran
d.)	bisa menjadi 30 anggota dan ditambah i pemimpin agar menjadi 30 oran ni t ni
d.)	b ira menjaui 30 anggora dan ditambah i pemimpin agar menjadi 50 oran ni + ni + ni + ni + ni + ni + ini + ini + ni

Figure 3. SKT-02 Answer

At the *Perception of generality* phase, shown in Figure 3, SKT-02 could recognize that the problem in the item was a pattern. SKT-02 showed this by stating that there is an addition of four to each line written with n = +4. SKT-02 was able to identify that each pattern always increases steadily. The pattern generalization strategy used by SKT-01 was counting. SKT-02 counted the number of squares, then realized that the number of squares corresponded to the order of the squad pattern. Answer shows that SKT-02 knew the next squad line pattern could be obtained using a certain rule.

Students with Moderate Mathematical Generalization Ability (SKS)

Ь.	Saloh	r	5,9	, 13,	5	21)	(h	uccun	17		1				
c.	benan	•	5.9	63,	17,	21,2	5. 2	9)	5				1	1	
D.	-	sala	h,	5+	9 +	137	N+	21 +	-25 +	29	73	3 +	37	+	4

Figure 4. SKS-01 Answer

At the *Perception of generality*, shown in Figure 4, SKS-01 could recognize that the problem in the question item was a pattern. It was knowing this from shown by SKS-01, who mentioned the sentence fixed formation. The word "formation" used by SKS-01 is slightly ambiguous because a fixed formation means that the number of team members does not change. This answer also shows that SKS-01 knows that the next squad lineup pattern can be obtained using a certain rule but has yet to identify the rule that applies to the problem fully. SKS-01 still needs to meet the indicators of this early generalization phase fully.

SKS-01 could describe the rule or pattern numerically at the *Expression of the generalization* phase. SKS-01 showed this with the answer to question point b. SKS-01 found that the number of members in the fifth team's line pattern was 21, not 17. SKS-01 identified the pattern in the problem, then used the identification result to determine the next term. Also seen in SKS-01's answer, he could numerically describe the rules in the problem. So that in question point b, SKS-01 can find the number of members in the fifth team's line pattern. Next, the results of SKS-02's work in completing the mathematical generalization test is presented in Figure 5.

1.	a. Benar
-	Alosan = karena pola bilangannya sama
	b. Salah
102	Alason = karena setiap pola ditambah 9, mk jika ket=5×9=20, dikumbah 5 di tendah gailuz
	c. Benar
	Alasan= karena pola ke-7=29
	d. Salah
	Aloron = 5+9+13+17+21+25+29+32+37+91=1 = 230

Figure 5. SKS-02 Answer

In the first phase, *Perception of generality*, shown in Figure 5, SKT-02 could recognize that the problem in the item was a pattern. It was known this from shown by SKS-02, who mentioned that the number of pattern sentences was the same. SKS-02 should have mentioned what pattern or rule was used in this problem. In the next work item, SKS-02 pinned down the applicable pattern as "any pattern plus 4". From this reasoning, since SKS-02 could fully identify the rules that applied to the problem at hand, it can be concluded that SKS-02 knew that the next team's lineup pattern was obtained using a particular rule.

SKS-02 could describe the rule or pattern numerically at the *Expression of the generalization* phase. SKS-02 showed this with the answer to question point b. SKS-02 identified that the pattern used to solve this problem was the addition of four members. However, SKS-02 found the wrong final answer, even though the pattern was correct. In his work, SKS-02 needed to be more careful in calculating. SKS-02 has yet to fulfill the second phase in this generalization.

Students with Low Mathematical Generalization Ability (SKR)

At the *Perception of generality*, shown in Figure 6, SKR-01 could recognize that the problem in the question item was a pattern. However, SKR-01 needed to be corrected in recognizing the type. Known this shown by SKR-01 mentioning the sentence "geometric sequence consisting of ratios." SKR-01 should have mentioned the pattern or rule used in this problem. SKR-01 pinned the applicable pattern as b = 4 in the next work point. For this reason, it can be concluded that SKR-01 knows that the pattern of the next team line is obtained using a certain rule because it can fully identify the rules that apply to the problem at hand.

a.) benar, karana pada pola tersi	ebut miruparan pola dan deret geometri
yang brdini atas Rasio yang arh	inga bilangan telap/sama,
b) $U_n = q + (n-1).b$	
45 = 9 + (5-1).9	- C
$u_{5} = 9 + 4.9$	
US = 32 , Salah seharusnya	jawabannya a
(1) = (1) + (1)	Suku Parlama A Varaan - 24
$h = \frac{1}{2}$	Kedua = 8 Ketwinh = 18
= 10 62.4+10-17.4	Kenga = 12 Kedelapon = 32
2	keempat= 16 kesembilan = 36
= 5 + (8+10 7 .4	Kelima 20 Kesepuluh : 40
	Jumcah: 210
= 5 + 18.9	- <i>I</i> /
: 5+7 2	(maka jawabannya sulah) yang tepat adalah
= 77	210

Figure 6. SKR-01 Answer

Base on Figure 6 show SKR-01 could describe the rule or pattern numerically at the *Expression of the generalization* phase. It is known that SKR-01 shows this with the answer to question point b. SKR-01 wrote the appropriate formula. However, SKR-01 needed to be corrected in identifying the initial term. The question stated that the initial term was five people, but SKR-01 wrote four people. It can occur due to the subject's misconception of language by the research results. Furthermore, SKS-01 must be corrected and corrected in the calculation process, causing the wrong final answer. SKR-01 still needs to fulfill the Expression of the generalization phase.

At the *symbolic expression of generalization* phase, SKR-01 could not produce a rule in general. SKR-01 shows this with the answer to question point c. SKR-01 wrote, "The pattern increases by 4, so a multiple of 4". SKR-01 could not answer the question correctly, so he could not find the right reason. Furthermore, at the *Manipulation of the generality* phase, SKR-01 was unable to solve the problem by using the results of his generalization. SKR-01's results had to show the correct answer regarding the total number of members. So SKR-01 did not fulfill the *symbolic expression of generalization* phase and *Manipulation of generality*. Next, the results of SKS-02's work in completing the mathematical generalization test is presented in Figure 7.

Figure 7. SKR-02 Answer

At the *Perception of generality* phase, shown in Figure 7, SKR-02 could recognize that the problem in the question item was a pattern. It was shown by SKR-02 mentioning the sentence "the formation increases by 4". SKR-02 mentioned what kind of pattern or rule was used in this problem, namely increasing by 4. For this reason, it can be concluded that SKR-02 knew that the next team's lineup pattern was obtained using a certain rule because he could fully identify the rules that applied to the problem.

Discussion

Students with High Mathematical Generalization Ability (SKT)

SKT-02 could describe the rule or pattern numerically at the *Expression of the generalization* phase. It was shown by SKT-02's answer to question point b. SKT-02 found that the number of members in the fifth team line pattern was 21. SKT-02 identified the pattern in the problem, then used the identification result to determine the next term. Also seen in SKT-02's answer, he could numerically describe the rules in the problem. So that in question point b, SKT-02 could find the number of members in the fifth team's line pattern. This is in line with previous study state that generalization ability need to be encouraged in learning mathematics, students with generalization abilities can expand and transfer knowledge to practical knowledge (Hayuningrat & Rosnawati, 2022).

At the *Symbolic expression phase of generalization*, SKT-02 was able to produce a general rule. SKT-02 showed this with the answer to question point c. SKT-02 wrote that the pattern of the seventh team's lineup was 29. So when connected to the problem, SKT-02 was able to find the right reason. Furthermore, SKT-02 solved the problem by using the results of his generalization in the *Manipulation of the generality* phase. SKT-02 applied the rule or pattern he had found to answer the question in point d. SKT-02 could show the correct answer that the total number of members involved was 230. It was known from SKT-02's answer, which is elaborated on one by one based on the results of his identification in the initial phase. Based on the results written by SKT-01 and SKT-02, subjects with high mathematical generalization ability category have used all phases of Mason's generalization well. Students with high mathematical generalization, *Symbolic expression of generalization*, and *Manipulation of generality* (Domike & Odey, 2014; Iswara et al., 2022).

Students with Moderate Mathematical Generalization Ability (SKS)

At the *Symbolic expression phase of generalization*, SKS-01 was able to produce a rule in general. SKS-01 showed this with the answer to question point c. SKS-01 wrote that the required squad line pattern was 29. So that when connected to the problem, SKS-01 could find the right reason. Furthermore, at the *Manipulation of the generality* phase, SKS-01 solved the problem by using the results of his generalization. SKS-01 applied the rule or pattern he had found to answer the question in point d. SKS-01 showed the correct answer that the total number of members involved was 230 people. It was knowing this from SKS-01's answer, which elaborates one by one based on the results of his identification in the previous initial phase. However, in SKS-01's work, there was doubt written by SKS-01 because he wrote ">200" at the end of his answer. The researcher still needs to do further confirmation of SKS-01's work. It is supported by study state that complex mathematical tasks such as problem solving are an ideal way to provide students opportunities to develop higher order mathematical processes such as representation, abstraction, and generalization (Sriraman, 2003).

At the *symbolic expression of generalization* phase, SKS-02 was able to produce a general rule. SKS-02 showed this with the answer to question point c. SKS-02 wrote that the required squad line pattern was 29. So that when connected to the problem, SKS-02 could find the right reason. Furthermore, at the *Manipulation of the generalization* phase, SKS-02 solved the problem by using the results of his generalization. SKS-02 applied the rule or pattern he had found to answer the question in point d. SKS-02 showed the correct answer that the total number of members involved was 230 people. It was known from SKS-02's answer, which elaborates one by one based on the results of his identification in the initial phase. Based on the work written by SKS-01 and SKS-02, subjects with moderate mathematical generalization

ability category have used only some of Mason's generalization phases. Students with moderate mathematical generalization ability only meet three phases: *Expression of generalization, Symbolic expression of generalization,* and *Manipulation of generality* (Fiangga et al., 2019; Malloy-Weir et al., 2016).

Students with Low Mathematical Generalization Ability (SKR)

SKS-02 could describe the rule or pattern numerically at the *Expression of the generalization* phase. From the answer question point b, SKS-02 was able to identify the pattern used to solve the problem, then used the identification results to determine the next term. Also seen in SKS-02's answer, he could numerically describe the rules in the problem. So that in question point b, SKR-02 can find the number of members in the fifth team's line pattern.

At the *Symbolic expression of generalization* phase, SKR-02 could not produce a general rule. SKR-02 showed this by not answering question point c. Likewise, with the *Manipulation of generality phase*, SKR-02 did not write the answer at point d, so SKR-02 could not solve the problem using the results of his generalization. So SKR-02 did not fulfill the symbolic expression of generalization phase and *Manipulation of generality*. Based on the work written by SKR-01 and SKR-02, subjects with low mathematical generalization ability category have used only some phases of Mason's generalization. Students with low mathematical generalization ability only meet two phases: *Perception of generalization* and *Expression of generalization* (Magiera & Zambak, 2021; Setiawan et al., 2020).

This kind of research can help educators better understand how students generalize in solving math problems. The implication is that teachers can develop more effective teaching strategies to help students develop mathematical generalization skills. The findings from this research can contribute to the development of better mathematics curricula that promote deeper conceptual understanding and mathematics generalization skills. However, this research also has limitations. One of the limitations in this research is time and resources which may limit the scope of the research and the number of student samples that can be included in the research.

4. CONCLUSION

Based on the results and discussion of data analysis showed the student's mathematical generalization ability of SMP on number patterns were obtained: (1) 8 students had high mathematical generalization abilities, (2) 15 students had moderate abilities, and (3) 5 students had low abilities. Students with high mathematical generalization ability were able to meet all of Mason's generalization indicators, students with moderate mathematical generalization ability met at least three indicators, and students with low mathematical generalization ability only met two of Mason's generalization indicators. Teachers can improve the quality of learning by effective teaching related to the concepts of pattern generalization and giving special attention to student weaknesses.

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