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How Teachers Encourage Students' Mathematical Reasoning during the Covid-19 Pandemic?

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ABSTRAK

Kemampuan penalaran matematis siswa dapat berkembang tergantung pada keterlibatan guru di kelas. Masalah yang sering terjadi di kelas adalah belum tersusun rapi tindakan guru dalam mendorong penalaran matematis siswa di dalam pembelajaran matematika. Artikel ini bertujuan untuk mengamati tindakan yang dilakukan sekelompok guru matematika dalam mendorong kemampuan penalaran matematis siswanya. Jenis penelitian ini adalah penelitian kualitatif dengan data yang dikumpulkan dari rekaman pembelajaran di kelas. Subjek penelitian adalah tiga guru matematika SMP. Guru mengamati bagaimana siswa bernalar selama 12 sesi saat mereka belajar dan menyelesaikan pekerjaan rumah. Hasil pengamatan aktivitas penalaran matematis siswa, seperti membangun argumen yang masuk akal dan menarik kesimpulan yang benar, disimpulkan dapat memungkinkan guru menguji penalaran matematis siswa mereka. Temuan penelitian ini menunjukkan bahwa guru dapat menumbuhkan penalaran matematis siswa dengan menerapkan strategi sebagai berikut: (a) memberikan intervensi pembelajaran yang tepat; (b) menetapkan tugas-tugas terbuka tanpa metode solusi; dan (c) membina komunitas belajar yang menghargai penalaran dan konstruksi gagasan bersama.

ABSTRACT

Students' mathematical reasoning abilities can develop depending on the teacher's involvement in the classroom. The problem that often occurs in the class is that the teacher's actions have not been neatly arranged to encourage students' mathematical reasoning in learning mathematics. This article aims to observe the actions taken by a group of mathematics teachers in encouraging students' mathematical reasoning abilities. This type of research is qualitative research with data collected from learning recordings in the classroom. The research subjects were three junior high school, mathematics teachers. The teacher observed how students reasoned during 12 sessions as they studied and completed homework. The results of observing students' mathematical reasoning activities, such as building reasonable arguments and drawing correct conclusions, were concluded to enable teachers to test their students' mathematical reasoning. The findings of this study indicate that teachers can foster students' mathematical reasoning by applying the following strategies: (a) providing appropriate learning interventions; (b) assigning open tasks without solution methods, and (c) fostering a learning community that values reasoning and the construction of shared ideas.

1. INTRODUCTION

When solving problems, students must be able to construct arguments and guarantee that the reasons presented are legitimate and correct and capable of leading to the correct conclusions. Students must have the ability to construct arguments from the moment they begin learning mathematics (Bleiler et al., 2014; Segerby & Chronaki, 2018). Students' ability to construct arguments, on the other hand, does not always appear on its own. Mathematics teachers in the classroom must play a role in encouraging pupils to construct valid arguments using mathematical reasoning activities (Dawkins & Roh, 2016; Hilton et al., 2016). Teachers should try to include all pupils in the mathematical reasoning process. The problem that often occurs in the class is that the teacher's actions have not been neatly arranged to encourage

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students' mathematical reasoning in learning mathematics (Bleiler et al., 2014; Rumsey & Langrall, 2016; Sumpter, 2018). Teachers' engagement in fostering students' mathematical reasoning can concentrate on how teachers track the types of reasoning that high school students exhibit when working on open assignments and evolve in solutions to middle school students' assignments (Ayalon & Even, 2016; Mata-Pereira & da Ponte, 2017). This condition is connected to how the teacher addresses students' reasoning behaviour when performing mathematical problems (Hidayat & Prabawanto, 2018; Mueller et al., 2014). Following this, the teacher's actions in assisting students to improve their mathematical thinking are carried out mainly during the justification and generalization stages. The urgency of the teacher's actions and movements in encouraging students to reason mathematically is to ensure that the student's reasoning process is reasonable and valid. The framework for analysing how teachers act in encouraging students' mathematical reasoning uses Ponte's framework.

This research is relevant to previous research, showing the importance of mathematics teachers assigning learning activities and problem-solving tasks to help students improve their proportional reasoning skills (Kramarski, 2008; Rohati, Turmudi, et al., 2021). In addition, previous study offers suggestions on how to deal with the many practical problems in student learning in the classroom and describes ways to construct reasoning learning. Many studies have shown that teachers must generate meaningful and enjoyable mathematics instruction for their students (Russo et al., 2020; Wilkinson et al., 2018; Yeh et al., 2019). Students must be directed to be active and critical in their mathematics learning through activities that create many valid arguments and ideas. Thus, teacher intervention is required in every mathematics class to direct students to provide arguments and be careful in solving issues that demand thinking (Gersten et al., 2009; Kilhamn et al., 2019; Yao & Manouchehri, 2020). The teacher's capacity to direct students to draw proper conclusions from each mathematical problem they answer . This study builds on study on how teachers might help pupils improve their mathematical reasoning skills (da Ponte & Quaresma, 2016; Garfield et al., 2008). Due to the ongoing Covid-19 pandemic, researchers are looking into how teachers might inspire students to learn using a variety of methods, including both offline and online learning (Branch, 2010; Buforn et al., 2014; Lin et al., 2017). According to previous study students want to improve their learning ability and manage it more effectively (Rohati, Marlina, et al., 2021). When students are given mathematical problems, including number patterns and object configurations, the teacher's efforts to intervene vocally in learning mathematics are revealed in this paper. The intervention in question is how the teacher encourages students to develop their ideas when completing tasks. In addition, the teacher can also direct the ideas given by a student to be well received by other students and justify the conclusions given by students properly.

The actions and movements observed by the teacher actions and movements consist of inviting and aiming to start a discussion. Another action is supporting/guiding and directing students in completing assignments through questions or observations that point (explicitly or implicitly) to a path they can follow. The third action is to inform/suggest, introduce information, make suggestions, present arguments, or validate student responses. The final action is challenging, requiring students to generate new representations, understand claims, make connections, or create reasoning or evaluations. The originality or novelty of this research in the way that learning about teachers' behaviors might support students' mathematical reasoning. Consequently, this study aims to monitor instructor behaviors intended to enhance students' mathematical reasoning in the classroom and enhance students' mathematical reasoning utilizing Ponte's theoretical concepts. The objectives are broken down into two parts, the first of which is to identify the steps taken by the teacher to support students use of Ponte's theory in their mathematical thinking. Second, to see students' reactions when the teacher has boosted their reasoning abilities.

2. METHOD

This descriptive approach was chosen to explore or photograph the teacher's actions in encouraging students' mathematical reasoning thoroughly, broadly, and deeply. This research was conducted at three public junior high schools in Jambi City The mathematical core of this topic includes number patterns and object configurations. To encourage students' reasoning, the teacher employs a behavioural framework established by, which highlights that student behaviour in the classroom is directly tied to the teacher's mathematics behaviour (van der Sandt, 2007). As a result, before witnessing and analysing students' mathematical reasoning when learning mathematics, it is critical to monitor teacher conduct in fostering students' mathematical reasoning. The procedure of gathering data, exposing data, correcting, and condensing data, and deriving research results are all processes followed in this study is show in **Figure 1**.

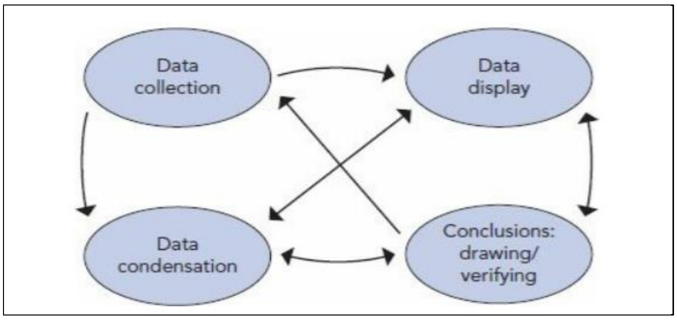
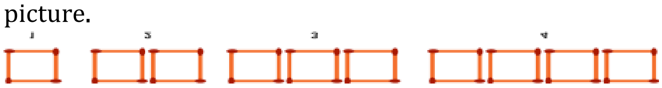


Figure 1. Interactive Model is a Data Analysis Component

The study was conducted at three grade 8 junior high schools in Jambi City. The research subjects were three junior high school, mathematics teachers. One maths teacher represents each school. Each teacher delivered a learning intervention consisting of four one-hour sessions. The assigned activities are based on the researcher's interaction with the teacher about the students' ability and the students' condition while studying during the Covid-19 pandemic. The teacher uses a combination of face-to-face and online learning through Google Classroom and WhatsApp. This situation is not great because the Covid-19 pandemic is not done yet.

The teacher decides the criteria for the question's level of difficulty and adjusts it to the student's abilities and the learning experience that is expected to appear in the student when selecting the tasks to be given in class. Furthermore, the questions chosen must be capable of encouraging students to present reasons, justifications, and the finishing process and form conclusions **Table 1**. According to the teachers, since the COVID-19 pandemic, student endurance in learning has decreased. The teacher must be able to assign the appropriate questions based on aptitude. The next stage only assigns more difficult questions for students to solve.

Tabel 1. Mathematical Tasks in the Teaching Action

Session	Learning Experience	Sample Task
1-3	Observe the pattern on a number sequence.	Find the number of circles in the 10th and 100th patterns? 
4-6	Determine the next term of a number pattern by generalizing the previous number pattern.	Determine the sequence pattern of numbers 5, 10, 8, 13, 11, 16, 14 and determine the successive two terms?
7-9	Generalize the pattern of number sequences into an equation.	Matchsticks are arranged in an arrangement as shown in the picture.  If the pattern continues, how many matchsticks are needed for the 10th picture? Explain how you found out! Find the rule for finding the number of toothpicks used in each image. Explain in words or write the formula!
10-12	Recognize the various number sequences.	Write the number of Pascal's triangle continued in the picture until the order of 10! 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1

The analytical procedure is carried out to detect how the teacher stimulates students' mathematical reasoning by watching the teacher's behaviours during learning and relates to the assignments assigned to students by the teacher. Data analysis was performed by analysing the teacher's learning videos, including interview transcripts and student assignment answers.

3. RESULT AND DISCUSSION

Result

In this section, the research findings are divided into two parts, the first of which concerns the types of actions made by the teacher to encourage students' mathematical reasoning using Ponte's theory. Second, a discussion of student responses based on the desired learning experience in which the teacher has previously intervened in the reasoning process.

Types of Teacher Action in Encouraging Students' Mathematical Reasoning

In the inviting action, the teacher initiates the discussion by asking the students preliminary questions. The teacher uses prior knowledge to motivate students to seek and accumulate knowledge that can be used and is required to acquire the content to be studied. In session 3 of the number pattern content, the teacher reminded students about subtracting integers once again. This content provides the foundation for children to comprehend the distinction between two numbers. This content is essential when beginning to enter number pattern material, such as arithmetic sequences. Figure 2 depicts the display when the teacher activates the students' prior knowledge.

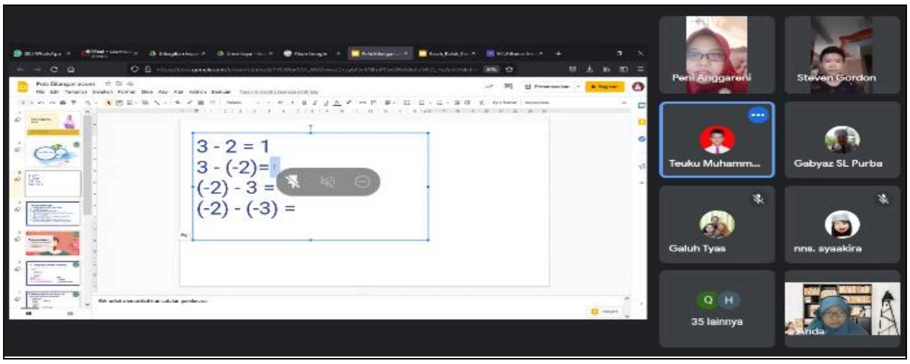


Figure 2. Teacher Display Invites Students for Discussion

In the Supporting/Guiding action, the teacher directs students through a task by asking questions or making observations that point (explicitly or implicitly) to a path they may take. In session 5, for example, the teacher instructs pupils to count the number of circles in the tenth pattern. After the students have responded with appropriate reasoning, the teacher instructs them to search for the 100th pattern. Then the teacher informs/suggests, introduces knowledge, gives ideas, presents arguments, or assesses student responses in informing/suggesting. For example, when students are figuring out the two successive terms of a numerical pattern, the teacher first provides the most straightforward problem. For example, the teacher might give a regular number pattern with the exact change in each subsequent pattern element. The teacher then presents a pretty tricky question. However, the teacher assists students by inviting them to study two separate rows, asking them to determine which of the two queries is accurate, and asking them to respond.

The teacher requires students to generate new representations, understand claims, make connections, or make reasoning or evaluations during the Challenging stage. The teacher instructs students to create visual representations of mathematical ideas by using tables to show the numbers formed. Furthermore, in the form of an image representation of the object's configuration, that becomes the context of the problem. In addition, the teacher requests that students display verbal representations of mathematical ideas by using words or phrases to explain, debate, describe, or illustrate them. The teacher reviews every argument presented by students to ensure that it is logical and sound. An overview of the emergence of each sort of teacher action and movement in encouraging students' mathematical reasoning through inviting, supporting, proposing, and challenging Table 2.

Tabel 2. Mathematical Tasks in the Teaching Action

School	Teacher Code	Session	Inviting	Supporting/ Guiding	Informing/ Suggesting	Challenging
I	TM-1	1	15%	56%	10%	12%

		2	12%	51%	15%	7%
		3	17%	37%	17%	10%
		4	20%	41%	12%	5%
II	TM-2	5	27%	46%	7%	12%
		6	29%	32%	17%	15%
		7	37%	27%	10%	24%
		8	32%	54%	27%	17%
III	TM-3	9	24%	39%	15%	22%
		10	20%	63%	20%	2%
		11	17%	29%	22%	10%
		12	15%	46%	17%	15%

Based on Table 2 the appearance of the teacher's activities in each learning session, the data is displayed as a percent. The teacher evaluates every argument presented by students to ensure that it is logical and correct. The teacher then instructs students to generalize the pattern of number sequences and object configuration sequences. Finally, the teacher encourages students to solve patterns and sequences of object configurations problems. The summarizes the description of the data is show in Table 3.

Tabel 3. Offline and Online Learning Platform, Category of Teacher Action, and Examples

Session	Offline learning	Online Learning			Category	Example
	Limited face-to-face learning	**Via Whats App**	**Via google clasroom**	**Via Zoom Meeting /Google Meet**		
1-4	yes	yes	no	yes	Inviting	Can you give an example of something you have observed that has a pattern in your daily life?
					Supporting/ Guiding	What do you think that the number of circles for the 10th pattern is 20? Do you agree?
					Informing/ Suggesting	Yes, it is according to the steps chosen to find the 100th pattern. Can Fatiah continue?
					Challenging	Is there a different strategy for determining the 100th pattern to make it easier?
5-6	no	yes	yes	no	Inviting	How many matchsticks are needed for the 10th picture?
					Supporting/ Guiding	It was interesting what Shashi said. Do you agree with his opinion?
					Informing/ Suggesting	The pattern you found is correct. Can you find the successive two terms of the number pattern?
					Challenging	I do not understand and have doubts.
7-9	no	yes	yes	yes	Inviting	Can you tell the difference between two numbers before entering the content of number patterns in the form of arithmetic sequences? Pay attention to the following problems!
					Supporting/ Guiding	Ok, do you agree with Briana's opinion? Is it an arithmetic

	Offline learning	Online Learning				
Session	Limited face-to-face learning	Via Whats App	Via google clasroom	Via Zoom Meeting /Google Meet	Category	Example
						sequence where the difference between two consecutive numbers is the same?
					Informing/ Suggesting	That's right, and the tenth pattern is 40. How about the nth pattern? Can the 4n formula determine it?
					Challenging	I am not sure if this is true
10-12	yes	yes	no	no	Inviting	What do you think the tenth line will be?
					Supporting/ Guiding	Convince me that what you say is true, Nabila?
					Informing/ Suggesting	Can you continue Pascal's number pattern to the 10th row? Why do you believe you have to add up every row of Pascal's triangle you find?
					Challenging	Why do you believe you have to add up every row of Pascal's triangle you find?

Base on Table 3 show the research's subsequent discovery is a set of samples of phrases uttered by the teacher throughout each learning session, which are linked to categories of how the teacher stimulates students' mathematical reasoning using Ponte's theory. The teacher's behaviours were observed in both online and offline learning environments. Offline, the researcher watched the teacher's actions in the classroom first-hand and then again through video recordings. In terms of online learning, researchers looked at zoom and Google Meet and WhatsApp, and Google Classroom. The researcher present several excerpts from interviews that occurred when the teacher responded to student responses with actions to lead, direct, and suggest students when giving arguments in the reasoning process when completing the given assignment in the findings of this study. The interview excerpt section is given in the discussion, accompanied by student answers of this paper.

Sample Answers and Student Responses for each Task

Students display visual representations in little circles to calculate the 10th pattern based on their answers in the first challenge is show in Figure 4.

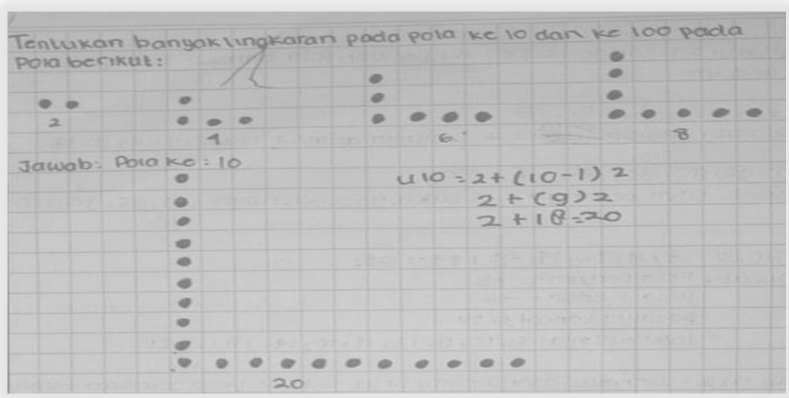


Figure 4. 10th Number Pattern Student Answers

Base on Figure 4 the teacher gives them instructions on figuring out how many circles there are by portraying them in an even number pattern (2, 4, 6, 8, ...). The teacher then instructs pupils to determine the first term, the difference, and then the larger term, Furthermore, when students are asked to determine the pattern of the 100th row, they can do so quickly, as illustrated in **Figure 5**.

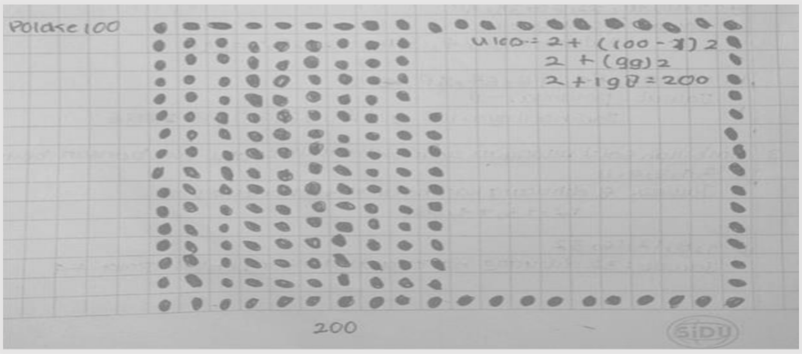
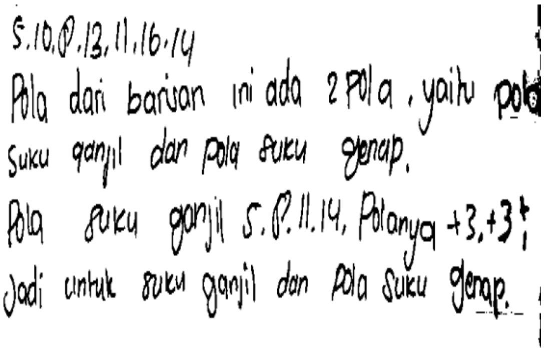


Figure 5. 100th Number Pattern Student Answers

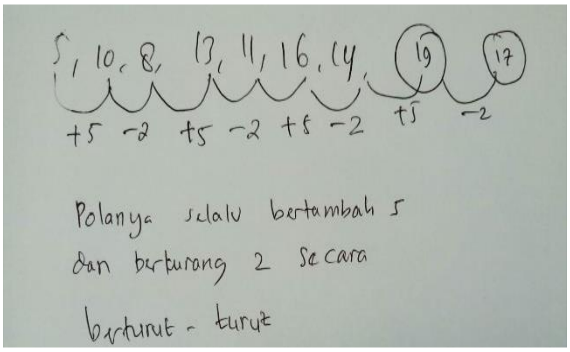
Most students still offer incorrect replies when identifying the following number from the supplied pattern. For example, in Figure 6, Briana writes an argument about determining the pattern of numbers.



Translate:
There are two patterns of this number: the odd tribal pattern and the even tribal pattern. Odd pattern consists of 5, 8, 11, and 14.
The pattern increases by 3, increases by 3. so for odd terms and even patterns.

Figure 6. Incorrect Student Answers

Base on Figure 6, Briana claims that the sequence has two patterns: odd number patterns and even number patterns. Briana's reasoning also inaccurately, specifically that the odd tribal pattern is 5, 8, 11, 14 when 14 is an even number. After reviewing the response, the teacher instructed Briana to attempt to determine the correct pattern from the given number pattern. Briana was able to discover the correct pattern after being given directions, as illustrated in Figure 7.



Translate:
The pattern always increases by 5 and decreases by 2 in a row

Figure 7. Student's Corrected Answers

Base on **Figure 7**, the teacher then instructs students to look for patterns that will bring them to the nth pattern. Students can deduct from the image that the nth pattern in the sequence of item configurations presented in assignment 3 is 4 × n. Then pupils performing generalizations. Students first typed the first pattern as 4, the second pattern as 8, the third pattern as 12. An example of recognizing the various number sequences is show in **Figure 8**.

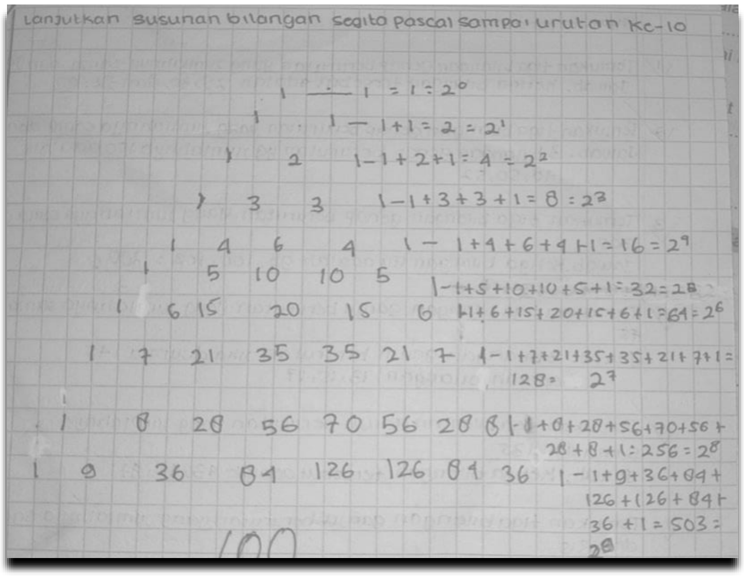


Figure 8. Students' Answers to the Pascal's Triangle Number Pattern

Base on **Figure 8** is an example of recognizing the various number sequences. Pascal number patterns that students can understand. When asked to determine the 10th line, students were able to write it smoothly. Students are also able to determine the nth number using the 2^{n-1} equation. Results are the main part of scientific articles, containing: final results without data analysis process, hypothesis testing results. Results can be presented with tables or graphs, to clarify the results verbally.

Discussion

Similar studies conducting in implementing learning interventions to increase students' mathematics understanding abilities (Yao & Manouchehri, 2020). We observed interactions between students and teachers in this study through either limited face-to-face instruction or virtual classrooms. The four categories of acts performed by the teacher are based on the philosophy of mathematical reasoning, which emphasizes the necessity of arguments that must be true before reaching conclusions and generalizations (Magiera & Zambak, 2021; Tatton-Brown, 2019; Zazkis & Villanueva, 2016). Students must be able to present the appropriate argument. Toulmin was long ago reminded that an accountable guarantee must back up a correct argument (Erduran, 2018; Mata-Pereira & da Ponte, 2017). In this study, observations made on the teacher's activities in fostering students' mathematical reasoning were thought to be very important. Because Indonesia is currently battling to escape the Covid-19 pandemic, most students are experiencing learning loss (Stylianou et al., 2015; Zhen et al., 2016). We also encountered difficulties performing observations because the setting of limited face-to-face learning activities and virtual classrooms was frequently changed. However, the data demonstrate that teachers are still making efforts to promote pupils' mathematical reasoning ability to grow (Corneli et al., 2019; Hallström & Schönborn, 2019; Komatsu & Jones, 2020). Prior to the covid-19 pandemic, a previous study emphasized the necessity of teachers promoting pupils' quantitative reasoning ability. Teachers' increased focus on fostering students' mathematical reasoning motivates teachers to continue intervening and taking action in mathematics learning in their classrooms as a whole.

In addition, the results of students' reasoning in mathematical reasoning can also be seen from the responses given to the four questions given. Written answers and student responses in learning are the effects of teacher activities in fostering students' mathematical reasoning in class (Bragg & Herbert, 2017; Timmermana et al., 2011; Vale et al., 2019). The study results show examples of students' answers and responses for each given task. Students can observe the pattern on a number sequence, determine the next term of a number pattern sequence, generalize the pattern of number sequences into an equation, and recognize the various number sequences. This ability is to the expected learning objectives in the number pattern material in grade 8. In addition, generalizations that appear in students' verbal answers and responses are a significant indicator of mathematical reasoning (Mumcu & Aktürk, 2017; Oslington et al., 2020). This research is also in line with research conducted by previous study. However, in this study, the students observed were junior high school students, while this research examined students aged five years (Csanadi et al., 2021; Göhner & Krell, 2020). It observes how what actions teachers take to provide opportunities for all students to learn and communicate their mathematical reasoning in a way that makes sense. The impact of teachers developing math practice with five-year-old students suggest that students can critically reason and engage in math practice. This research is also in line with research conducted by other study (Kooloos et al., 2021; Veldhuis & van den Heuvel-Panhuizen, 2020). The results show how teacher actions that encourage mathematical reasoning provide generalizations or justifications and teacher actions that support appropriate reasons.

According to the results of this study, teachers can nurture students' mathematical reasoning by starting a productive debate about the work at hand and how the teacher guides and directs students to put up the proper argument (Brodie, 2010; Loong et al., 2017; Yilmaz & Topal, 2014). The teacher can also take the appropriate action by assigning challenges to pupils who have not answered the questions and then improving the procedure correctly (Lambert & Schuck, 2021; Sgaravatti, 2018; Sullivan & Davidson, 2014). Furthermore, the teacher might assign problems that do not have a solution technique to motivate students to figure out how to solve them. When completing activities, teachers can also create ideas with students and encourage all students to work together in class. Inviting, supporting, and encouraging students are examples of teacher activities. Teacher behaviours such as inviting, supporting/guiding, informing/suggesting, and challenges must be strengthened to assist students' mathematical reasoning abilities to improve, especially during the ongoing Covid-19 pandemic (Aaron & Herbst, 2015; Csanadi et al., 2021; Mueller et al., 2014). The novelty of this paper lies in the teacher's actions in supporting students' mathematical reasoning in both face-to-face and online learning. The teacher's actions are intended to improve students' mathematical reasoning in the classroom and improve students' mathematical reasoning through Ponte's theoretical concepts. In addition, the discussion and flow of students' mathematical thinking presented in this paper also informs the reader about the importance of thinking and reasoning mathematically. This condition is in line with opinion that mathematical thinking is closely related to a student's ability to reason effectively (Rohati et al., 2022; Sawatzki et al., 2019). Although this study reports only on the actions of three mathematics teachers in learning mathematics in the classroom, the results can provide examples and illustrations for other teachers to encourage their students' mathematical reasoning. Other teachers can do the kinds of actions and talk they can promote in their classrooms to practice their students' mathematical reasoning. Further research is needed to examine the results of other classes on students' mathematics achievement at different levels.

4. CONCLUSION

This study investigated how teachers inspire students' mathematical reasoning in both offline and online classroom settings. The teacher's actions can include starting a constructive discussion, providing information, supporting, and suggesting, introducing information, presenting arguments, or validating student responses. Furthermore, teachers might set tasks that require students to construct new representations, under-stand claims, make connections, or develop reasoning or judgments. According to the findings of this study, teachers can grow students' mathematical reasoning by providing appropriate learning interventions, assigning tasks without solution methods, and fostering a community that values reasoning and the construction of shared ideas.

5. REFERENCES

Aaron, W. R., & Herbst, P. G. (2015). Teachers' perceptions of students' mathematical work while making conjectures: an examination of teacher discussions of an animated geometry classroom scenario. *International Journal of STEM Education*, *2*(1). https://doi.org/10.1186/s40594-015-0021-0.

Ayalon, M., & Even, R. (2016). *Factors shaping students ' opportunities to engage in argumentative activity*. *September 2014*, 575–601.

Bleiler, S. K., Thompson, D. R., & Krajčevski, M. (2014). Providing written feedback on students' mathematical arguments: proof validations of prospective secondary mathematics teachers. *Journal of Mathematics Teacher Education*, *17*(2), 105–127. https://doi.org/10.1007/s10857-013-9248-1.

Bragg, L. A., & Herbert, S. (2017). A" True" Story about Mathematical Reasoning Made Easy. *Australian Primary Mathematics Classroom*.

Branch, R. M. (2010). Instructional design: The ADDIE approach. In *Instructional Design: The ADDIE Approach*. https://doi.org/10.1007/978-0-387-09506-6.

Brodie, K. (2010). *Teaching Mathematical Reasoning in Secondary School Classrooms*. Springer New York Dordrecht Heidelberg London. https://doi.org/DOI 10.1007/978-0-387-09742-8.

Buforn, À., Fernández, C., Llinares, S., & Sánchez-matamoros, G. (2014). Characteristics of pre-service primary teachers ' noticing of students ' thinking related to fraction concept sub-constructs. *Proceedings of CERME10*, *1*, 2812–3031. https://hal.science/hal-01949061/.

Corneli, J., Martin, U., Murray-Rust, D., Rino Nesin, G., & Pease, A. (2019). Argumentation Theory for Mathematical Argument. In *Argumentation* (Vol. 33, Issue 2). Springer Netherlands. https://doi.org/10.1007/s10503-018-9474-x.

Csanadi, A., Kollar, I., & Fischer, F. (2021). Pre-service teachers' evidence-based reasoning during pedagogical problem-solving: better together? *European Journal of Psychology of Education*, *36*(1), 147–168. https://doi.org/10.1007/s10212-020-00467-4.

da Ponte, J. P., & Quaresma, M. (2016). Teachers' professional practice conducting mathematical discussions. *Educational Studies in Mathematics*, *93*(1), 51–66. https://doi.org/10.1007/s10649-016-9681-z.

Dawkins, P. C., & Roh, K. H. (2016). Promoting Metalinguistic and Metamathematical Reasoning in Proof-Oriented Mathematics Courses: a Method and a Framework. *International Journal of Research in Undergraduate Mathematics Education*, *2*(2), 197–222. https://doi.org/10.1007/s40753-016-0027-0.

Erduran, S. (2018). Toulmin's argument pattern as a "horizon of possibilities" in the study of argumentation in science education. *Cultural Studies of Science Education*, *13*(4), 1091–1099. https://doi.org/10.1007/s11422-017-9847-8.

Garfield, J. B., Ben-Zvi, D., Chance, B., Medina, E., Roseth, C., & Zieffler, A. (2008). Creating a Statistical Reasoning Learning Environment. In *Developing Students' Statistical Reasoning* (pp. 45–63). Springer Science+Business Media B.V. https://doi.org/10.1007/978-1-4020-8383-9_3.

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). Assisting Students Struggling with Mathematics:Response to Intervention (RtI) for elementary and middle schools. *What Works Clearinghouse*, 1–98.

Göhner, M., & Krell, M. (2020). Preservice Science Teachers' Strategies in Scientific Reasoning: the Case of Modeling. *Research in Science Education*, *Osborne 2013*. https://doi.org/10.1007/s11165-020-09945-7.

Hallström, J., & Schönborn, K. J. (2019). Models and modelling for authentic STEM education: reinforcing the argument. *International Journal of STEM Education*, *6*(1). https://doi.org/10.1186/s40594-019-0178-z.

Hidayat, W., & Prabawanto, S. (2018). Improving students' creative mathematical reasoning ability students through adversity quotient and argument driven inquiry learning. *IOP Conf. Series: Journal of Physics: Conf. Series*, 1–6. https://doi.org/doi :10.1088/1742-6596/948/1/012005.

Hilton, A., Hilton, G., Dole, S., & Hilton, A. (2016). *Promoting middle school students ' proportional reasoning skills through an ongoing professional development programme for teachers*. https://doi.org/10.1007/s10649-016-9694-7.

Kilhamn, C., Bråting, K., & Rolandsson, L. (2019). Teachers' arguments for including programming in mathematics education. *Preceedings of NORMA 20*, 169–176. https://www.diva-portal.org/smash/record.jsf?pid=diva2:1541426.

Komatsu, K., & Jones, K. (2020). Correction to: Interplay between Paper-and-Pencil Activity and Dynamic-Geometry-Environment Use during Generalisation and Proving. *Digital Experiences in Mathematics Education*, *6*(2), 144–144. https://doi.org/10.1007/s40751-020-00072-6.

Kooloos, C., Oolbekkink-Marchand, H., van Boven, S., Kaenders, R., & Heckman, G. (2021). Building on student mathematical thinking in whole-class discourse: exploring teachers' in-the-moment decision-making, interpretation, and underlying conceptions. *Journal of Mathematics Teacher Education*, *0123456789*. https://doi.org/10.1007/s10857-021-09499-z.

Kramarski, B. (2008). Promoting teachers' algebraic reasoning and self-regulation with metacognitive guidance. *Metacognition and Learning*, *3*(2), 83–99. https://doi.org/10.1007/s11409-008-9020-6.

Lambert, R., & Schuck, R. (2021). "The Wall Now Between Us": Teaching Math to Students with Disabilities During the COVID Spring of 2020. *Asia-Pacific Education Researcher*, *30*(3), 289–298. https://doi.org/10.1007/s40299-021-00568-8.

Lin, Y. W., Tseng, C. L., & Chiang, P. J. (2017). The Effect of Blended Learning in Mathematics Course. *Eurasia Journal of Mathematics, Science and Technology Education*, *13*(3), 741–770. https://doi.org/10.12973/eurasia.2017.00641a.

Loong, E. Y., Vale, C., & Bragg, L. A. (2017). Tracking change in primary teachers ' understanding of mathematical reasoning through demonstration lessons. *Mathematics Teacher Education and Development*, *19*(1), 5–29. https://eric.ed.gov/?id=EJ1152733.

Magiera, M. T., & Zambak, V. S. (2021). Prospective K-8 teachers' noticing of student justifications and generalizations in the context of analyzing written artifacts and video-records. *International Journal of STEM Education*, *8*(1). https://doi.org/10.1186/s40594-020-00263-y.

Mata-Pereira, J., & da Ponte, J. P. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics*, *96*(2), 169–186. https://doi.org/10.1007/s10649-017-9773-4.

Mueller, M., Yankelewitz, D., & Maher, C. (2014). Teachers Promoting Student Mathematical Reasoning. *Investigations in Mathematics Learning*, *7*(2), 1–20. https://doi.org/10.1080/24727466.2014.11790339.

Mumcu, H. Y., & Aktürk, T. (2017). An analysis of the reasoning skills of pre-service teachers in the context of mathematical thinking. *European Journal of Education Studies*, *3*(5). https://doi.org/10.46827/ejes.v0i0.652.

Oslington, G., Mulligan, J., & Van Bergen, P. (2020). Third-graders' predictive reasoning strategies. *Educational Studies in Mathematics*, *104*(1), 5–24. https://doi.org/10.1007/s10649-020-09949-0.

Rohati, Marlina, & Khidayatul, I. (2021). The Views of High School Students During The Covid-19 Period on Learning Mathematics. *IJECA (International Journal of Education & Curriculum Application)*, *4*(2), 64–74. https://doi.org/10.31764/ijeca.v4i2.5139.

Rohati, R., Kusumah, Y. S., & Kusnandi, K. (2022). The development of analytical rubrics : An avenue to assess students ' mathematical reasoning behavior. *Cypriot Journal of Educational Sciences*, *17*(8), 2553–2566. https://doi.org/https://doi.org/10.18844/cjes.v17i8.7043.

Rohati, R., Turmudi, T., & Kusnandi, K. (2021). Students' proportional reasoning in mathematics through Covid-19 pandemic context. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(3), 1670. https://doi.org/10.24127/ajpm.v10i3.3873.

Rumsey, C., & Langrall, C. W. (2016). Promoting mathematical argumentation. *Teaching Children Mathematics*, *22*(7), 412–419. https://doi.org/10.5951/teacchilmath.22.7.0412.

Russo, J., Bobis, J., Sullivan, P., Downton, A., Livy, S., McCormick, M., & Hughes, S. (2020). Exploring the relationship between teacher enjoyment of mathematics, their attitudes towards student struggle and instructional time amongst early years primary teachers. *Teaching and Teacher Education*, *88*, 102983. https://doi.org/10.1016/j.tate.2019.102983.

Sawatzki, C., Downton, A., & Cheeseman, J. (2019). Stimulating Proportional Reasoning through Questions of Finance and Fairness. *Mathematics Education Research Journa*, *31*, 465–484. https://doi.org/10.1007/s13394-019-00262-5.

Segerby, C., & Chronaki, A. (2018). Primary students' participation in mathematical reasoning: Coordinating reciprocal teaching and systemic functional lin-guistics to support reasoning in the Swedish context. *Educational Design Research: An International Journal for Design-Based Research in Education*, *2*(1). https://doi.org/10.15460/eder.2.1.1150.

Sgaravatti, D. (2018). Experience and reasoning : challenging the a priori / a posteriori distinction. *Synthese*. https://doi.org/10.1007/s11229-018-1718-7.

Stylianou, D. A., Blanton, M. L., & Rotou, O. (2015). Undergraduate Students' Understanding of Proof: Relationships Between Proof Conceptions, Beliefs, and Classroom Experiences with Learning Proof. *International Journal of Research in Undergraduate Mathematics Education*, *1*(1), 91–134. https://doi.org/10.1007/s40753-015-0003-0.

Sullivan, P., & Davidson, A. (2014). *The Role of Challenging Mathematical Tasks in Creating Opportunities for Student Reasoning*. 605–612.

Sumpter, L. (2018). Grade 9 Students' Reasoning About Division of Fractions: What Are their Arguments Anchored in? *Students' and Teachers' Values, Attitudes, Feelings and Beliefs in Mathematics Classrooms*, 135–143. https://doi.org/10.1007/978-3-319-70244-5_13.

Tatton-Brown, O. (2019). Rigour and Intuition. *Erkenntnis*, *86*(6), 1757–1781. https://doi.org/10.1007/s10670-019-00180-9.

Timmermana, B. E. C., Strickland, D. C., Johnson, R. L., & Paynec, J. R. (2011). Development of a "universal" rubric for assessing undergraduates' scientific reasoning skills using scientific writing. *Assessment and Evaluation in Higher Education*, *36*(5), 509–547. https://doi.org/10.1080/02602930903540991.

Vale, C., Widjaja, W., Doig, B., & Groves, S. (2019). Anticipating students' reasoning and planning prompts in structured problem-solving lessons. *Mathematics Education Research Journal*, *31*(1). https://doi.org/10.1007/s13394-018-0239-5.

van der Sandt, S. (2007). Research framework on mathematics teacher behaviour: Koehler and Grouws' framework revisited. *Eurasia Journal of Mathematics, Science and Technology Education*, *3*(4), 343–350. https://doi.org/10.12973/ejmste/75413.

Veldhuis, M., & van den Heuvel-Panhuizen, M. (2020). Supporting primary school teachers' classroom assessment in mathematics education: effects on student achievement. *Mathematics Education Research Journal*, *32*(3), 449–471. https://doi.org/10.1007/s13394-019-00270-5.

Wilkinson, L. C., Bailey, A. L., & Maher. (2018). Students' Mathematical Reasoning, Communication, and Language Representations_ A Video-Narrative Analysis _ Enhanced Reader.pdf. *Ecnu Review of Education*, *1*(3), 1–22. https://doi.org/10.30926/ecnuroe 2018010301.

Yao, X., & Manouchehri, A. (2020). Teacher interventions for advancing students' mathematical understanding. *Education Sciences*, *10*(6), 1–21. https://doi.org/10.3390/educsci10060164.

Yeh, C. Y. C., Cheng, H. N. H., Chen, Z. H., Liao, C. C. Y., & Chan, T. W. (2019). Enhancing achievement and interest in mathematics learning through Math-Island. *Research and Practice in Technology Enhanced Learning*, *14*(1). https://doi.org/10.1186/s41039-019-0100-9.

Yilmaz, Z., & Topal, Z. O. (2014). Connecting Mathematical Reasoning and Language Arts Skills: The Case of Common Core State Standards. *Procedia - Social and Behavioral Sciences*, *116*, 3716–3721. https://doi.org/10.1016/j.sbspro.2014.01.829.

Zazkis, D., & Villanueva, M. (2016). Student Conceptions of What it Means to Base a Proof on an Informal Argument. *International Journal of Research in Undergraduate Mathematics Education*, *2*(3), 318–337. https://doi.org/10.1007/s40753-016-0032-3.

Zhen, B., Weber, K., & Mejia-Ramos, J. P. (2016). Mathematics Majors' Perceptions of the Admissibility of Graphical Inferences in Proofs. *International Journal of Research in Undergraduate Mathematics Education*, *2*(1), 1–29. https://doi.org/10.1007/s40753-015-0010-1.