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Relearning the Calculus : Connecting it to Real-life

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Abstract

Developing students' abilities in solving real-life problems is one of the mathematics curriculum goals. Students may be motivated and interested as well as have good perceptions in learning mathematics when knowing the usefulness of mathematics for their life. However, teachers may have difficulties to connect some mathematics topics including calculus to students' real-life because of the nature of topic. Therefore, it is required to analyse how teachers can teach calculus and then connect it to real-life. This paper uses a library research method. Both primary and secondary sources are studied, identified, and analysed to know the teaching methods in teaching calculus and then connecting it to students' life. It is found that tough teachers will find some obstacles when teaching and connecting calculus to students' real-life, they may start to give more calculus problems to students. They also can use mathematics softwares to give sense of the usefulness of calculus in students' life.

Keywords: *R*real-life problems; calculus; the usefulness of calculus

INTRODUCTION

First and foremost, a student might be motivated to learn mathematics if they know when they can use it. In fact, students utilise mathematics in their life without knowing they have ever used it while some teachers do not transmit the usefulness of mathematics to their students including when teaching calculus. There is also a dilemma whether teaching mathematics for memorising or understanding. Most teaching and learning mathematics process that we encountered in the classroom focus on remembering owing to the number of formulas that students have to use to answer the exam. Teachers struggle more in teaching how students can apply formulas to answer the mathematics questions rather than explain when and why they use them. However, the common goal of school mathematics curriculum in some countries is developing students' problem-solving ability in solving real-life problems (Department of Education, 2013; MoNE, 2012; NCTM, 2000). It is in line with Sumirattana, Makanong & Thipkong (2017). They explained that the goal of teaching and learning mathematics in schools is to enhance students' mathematical literacy and their abilities in using and applying mathematical knowledge to solve problems or situations in real-life.

Other previous researchers established benefits of connecting mathematics to real-world while teaching mathematics concepts (Real-world connections). It can motivate students who like or dislike mathematics and create better attitudes (Gainsburg, 2008). It also changes students' negative perceptions of mathematics (Muijs & Reynolds, 2011). Moreover, teaching mathematics in real context can stimulate students in understanding the world around them as well as solving problems related to their life and employment (Blum, 2002; Patton, 1997). In their studies, Karakoç and Alacacı (2015) also discovered that most participants agreed that real mathematics connections helped to improve or develop students' motivation and interest, positive attitudes, mathematical process skills, conceptual and permanent learning, awareness of the choice of future careers, and ability to generalise mathematical ideas.

However, there is still less research about the topics that can be taught by using real-world connections as well as the ways teachers teach students using the real-world connections in the classroom (Gainsburg, 2008), including the study of Karakoç and Alacacı (2015). A half of participants in Karakoç and Alacacı’s study thought that the examples of real-world connections were complex and could be difficult as well as made abstract thinking difficult. They also believed that the limitation of using real-world connections was the lack of adequately equipped teachers and their reluctance as well as the nature of the mathematics topic. Therefore, the use of real-world connections in a teaching process could not be generalised. Possible places of the use of real-world connections depend on the topic and nature of the problem. Karakoç and Alacacı assert that teachers cannot practice all examples given by the participants from their study, but they less clearly explain what the characteristics of unpractical examples of real-world connections as well as which the examples that should not be used by teachers. Regarding calculus topic, Karakoç and Alacacı found that integration could be utilised for determining the area and volume of irregular shapes on the earth, but they did not analyse what kind of activities teachers can use for showing that they have made real-world connections.Thus, teachers require analysing relevant examples carefully when connecting mathematics to real-life including calculus as a part of the topics in mathematics.

Based on the explanation above, in this paper, I will find out some issues, those are: What is the meaning of connecting calculus with real-life problems?; What are the inhibiting factors in connecting calculus with real-life problems?; and How is the possibility of teaching calculus by connecting it to real-life problems in Indonesia later on?. To answer the questions, we have to know weather the definition of connecting calculus to real-life is using calculus to solve daily life problems. Then, Teachers’ competencies as well as the limitation of the teaching and learning sources and facilities may become the main obstacles for teachers to connect between calculus and real-life conditions. Regarding the ways to teach calculus, teachers may use some realistic questions related to the concept of calculus. However, to ensure the appropiate answers we need to analyse the literature more deeply.

METHOD

A library research method was used in this paper, which is "a research used library sources to get the data" (Zed, 2008, p.1). Research reports (primary sources) as well as books, journal, articles, and handbooks (secondary sources) were studied and then taking notes was done. These whole sources was indentified and analysed to find appropiate theories and to develop ideas related to how to teach calculus and then connect it to students' real-life. As a result, the theories regarding teaching calculus in the mathematics classroom can be found.

RESULTS AND DISCUSSIONS

The definition of connecting calculus to real-life problems

There are some terms regarding connecting something to the real-life problems. Gainsburg (2008) explained it by using a term of "real-world connections" which is all connections associated with "simple analogies, word problems, the analysis of real data, discussion of mathematics in society, representations of mathematics concepts, and mathematically modelling real phenomena" (p.200). Meanwhile, from another source with a bit different term, real-life connections are connecting mathematics that students learned in the classroom to their real activities (Mosvold, 2008).

Real-world connections (Karakoç & Alacacı, 2015; Gainsburg, 2008) and real-life connections (Mosvold, 2008), which are directly or indirectly related, are the solutions that might answer the question about what the meaning of connecting the mathematics including calculus that students' learn in the school and their experience outside the school (Mosvold, 2008). Using real-world connections also links to others issues: teaching in context, teaching the utility of mathematics related to context in the real world (Boaler, 1993); a concrete object, that is when we are able to visualise the object like reality (Wilensky, 1991); technological knowing, the ability for using mathematics in solving problems (Skovsmose, 1994); and mathematics manipulative, concrete objects that are used by students in learning mathematics (Uttal, Scudder, & DeLoache, 1997). All those allow students to connect their knowledge to experience in real life.

Moreover, making real-world connections is not exactly similar to the Realistic Mathematics Education (RME) approach (Karakoç & Alacacı, 2015). RME does not only mean connecting to the real-world but also making something real in students' minds by offering them situations. As a result, they are able to imagine problems coming from the real-world, fantasy world of fairy tales, or the formal of global mathematics (Van den & Drijvers, 2014). Meanwhile, using real-world connections refers to connecting students' mathematical concepts and symbols with their daily experience (Uttal, Scudder, & DeLoache, 1997).

Based on the explanation above, connecting calculus to real-life problems is applying calculus concept to solve students' daily problems based on their experience.

The inhibiting factors in connecting calculus to real life problems

There are many advantages of connecting mathematics to real-life problems (Gainsburg, 2008; Muijs & Reynolds, 2011; Blum, 2002; Patton, 1997), but there are still some criticisms which should be taken into account. *First,* is the fact that real-world connections in the classroom are not always related to the students' everyday life directly so that students do not necessarily connect mathematics to their real-life (Mosvold, 2008) such as radioactive decay, bacterial growth, and cryptography (Karakoç & Alacacı, 2015). Consequently, applying these connections will not necessarily give more meaning to them whereas the aim of using real-world connections is making mathematics more meaningful for all students (Yanik & Serin, 2016; Boaler 1993; Clarke & Roche, 2009). *Second,* direct connections with daily life may cause mathematics to be harder for the pupils to understand (Mosvold, 2008) because the real situations are likely complex (Karakoç & Alacacı, 2015). *Last,* the consideration of which one should be taught first, concepts or real-world connections. Using real-world connections can improve conceptual learning (Karakoç & Alacacı, 2015). However, students should understand mathematical concepts before connecting them to the reality (Gainsburg, 2008), because learning mathematics without conceptual understanding will be unproductive (Uttal, Scudder, & DeLoache, 1997). Thus, some factors may become difficulties for teachers to connect calculus to students' real-life problems are students and teachers' perceptions of real-life connections it self; the compexity of real-life situations faced by students, not all the topics can be related to students' real-life, and students' understanding regarding the concept of calculus.

On the other hand, there is an important suggestion for teachers about how to create an example of using real-world connections. They should consider in creating relevant examples because not all examples are practicable in real life context such as "complex number" (Karakoç & Alacacı, 2015). In fact, limitation of using real-world connections may guide curriculum developers, textbook writers, and mathematics teachers in high schools to select examples carefully because not all the real situations are relevant to pupils and mathematics curriculum (Clarke, 2009).

Karakoç and Alacacı (2015) found the limited sources of the examples of using real-world connections. Those limitations boost teachers to create their own examples for teaching mathematics in real context. For instance, a Japanese teacher taught congruence and similarity by giving the assignment for students to bring geometric figures with the same shape but the different size that students found at home, and then the teacher also brought some things and used these to introduce the topic (Mosvold, 2008). This fact shows, despite the fact that there are three sources of use of the real-world connections: textbook and other curricular materials, classroom teachers, and students (Yanik and Serin, 2016), teachers have a crucial role in deciding the sources of real life situations. Hence, the avalilable sources of teaching and learning calculus as well as teachers' competency for creating their own sources will affect the process of connecting calculus to students' real-life problems.

Another point is whether there might also be differences between how teachers connect mathematics to real situations and the real-world connection itself. Real-world connections mean all connections that teachers use for connecting what is taught in the classroom and daily activities. For example, numbers connected to calculation in shopping and trigonometry connected to architecture (Karakoç & Alacacı, 2015). Whereas how teachers connect mathematics to real life is a method that teachers apply for teaching using real-world connections (Mosvold, 2008). A noticeable evident for differences of teachers' ways in teaching using real-world connections, Netherlands (7^{th} grade in TIMSS 1999) had the highest number of using real life connections in which teachers focused on teaching problems from the textbook having some real-life contexts. In contrast, Japan (5^{th} grade) which had a lower number of using real-life connections instructed teachers to start teaching by connecting mathematics to the real life examples or abstract things that pupils had, and then went into the mathematical concept (Mosvold, 2008). This finding shows that even though both Dutch and Japanese teachers use real-world connections, they had a different strategy in teaching mathematics. Moreover, the way teachers teach their students is more likely to affect students' performance compared to just focusing on the number of real-world connections used. It is not a compulsory method for teachers to use real-world connections in teaching every topic. Therefore, teachers should focus on deciding an effective way to teach their students.

Is it possible for Indonesian Teachers to teach real-life connections in calculus topics?

Indonesia has adopted RME (Realistic Mathematics Education) from the Netherlands, known in Indonesia as PMRI (Pendidikan Matematika Realistik Indonesia) which correlates to teaching mathematics in real contexts and emphasises the application of mathematics (Sembiring, 2008). However, RME does not mean teachers have to involve the students in practical activities but create a meaningful learning activity so that students can imagine it like they do reality (Van den & Drijvers, 2014). Even though some previous researchers found that the implementation of PMRI in Indonesia had positive effects on students' mathematics achievement (Armanto, 2002; Fauzan, 2002), Indonesia has not made relevant PMRI curriculum materials (Sembiring, 2008). Therefore, Indonesia still needs to develop some resources related to the implementation of PMRI.

In addition, Indonesian mathematics teacher's ability itself will be a difficulty in implementing teaching mathematics in real contexts. One of their concerns is connecting mathematics to the real world to encourage students to deal with their daily life problems (Zamroni, 2000). However, some of them are only able to teach instrumental understanding (Skemp, 1976) in the classroom so that students learn calculus as formulas without realising how they use it. Students just follow teachers' instruction; memorising formulas, understanding the examples, and then solving the exercises. Undeniably, students own negative perspectives on mathematics, including the calculus, are due to this fact. Thus, teachers should find ways to improve these students' perspectives in order to enhance their understanding and achievement in mathematics.

Teachers can connect calculus to the real world by solving some real-life problems which Gravemeijer and Doorman (1999) describe as problems that use real-life contexts and can be imagined as daily experiences. Burkhardt (1981) also asserts that teachers can connect mathematics to real-life in the classroom through the collection of realistic problems that provide an opportunity for students to solve problems using their mathematical skills. Therefore, real-life problems can be used to enhance students' motivation and to develop reasoning as well as the problem-solving skills of students in learning mathematics (Karakoç & Alacacı, 2015). The teachers will be also able to make mathematics more meaningful for their students through real-life problems. Personally, there are some interesting real-life problems that Indonesian teachers can use such as Max box problem that can be connected to a manufacturer.

"Max box problem" is an example of a real life mathematics problem (Kemp, 2005). This problem is about a piece of paper which had side a in which we are instructed to make a box by cutting a square with side x from each of the four corners. The problem is how to find the value of x so that we could make the biggest box.

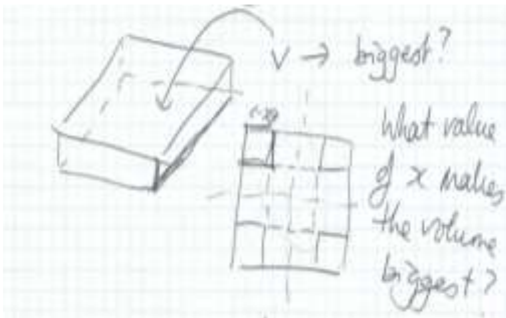

Figure 1. Max Box Problem

I tried to find the *x* value for creating the biggest box by doing some algebraic equations and finally, I obtained the pattern for finding the *x* value. Finding out the answer gave me an opportunity to relate it to the concept of differentiation. It was a new problem for me and when I searched on the internet, found it was popular in teaching and learning mathematics related to the calculus topic (STEM Learning, 2009). However, based on my own teaching experience, I found that Indonesian mathematics teachers rarely used this practical question while teaching the concept of differentiation. After trying to solve this problem, I agree that this problem should be taught in the classroom.

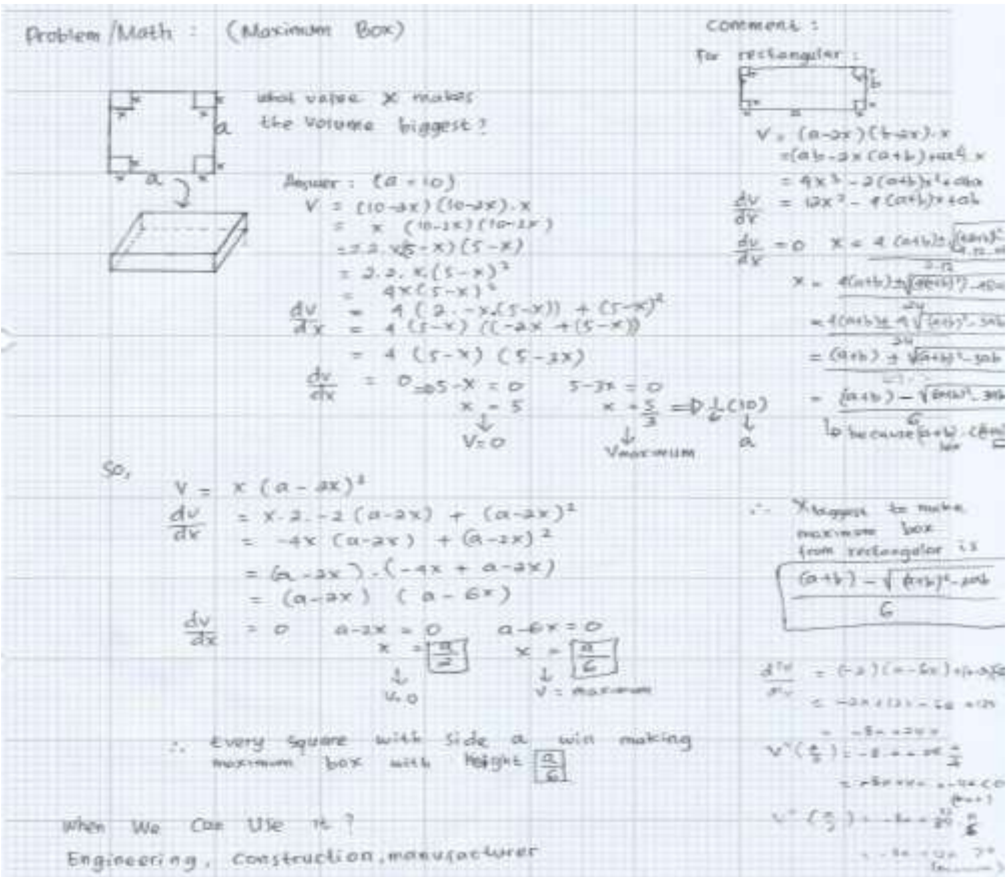

Figure 2. The Solution of Max Box Problem

Another possible method, teachers can give and explain some kinds of daily context questions from any sources found such as mathematics textbook while teaching (Mosvold, 2008), then discussing how they can solve the problems. Furthermore, I think that mathematics teachers may be able to explore any calculus questions on websites such as http://calculus.org/ that I would recommend as resources for finding real-life mathematics problems using the English language. However, teachers who are non-native-English speakers should be careful in understanding the meaning of the problems because there is a particular English term of mathematics that sounds unfamiliar or synonymous. For instance, it was confused when distinguishing between two words that seemed synonyms like capacity and volume. I firstly thought that those two words had similar meaning, however, capacity related to how much liquid held while volume related to how many materials needed (solid) in the container.

On the other hand, occasionally, overuse of real-world connections may complicate students to understand abstract concepts (Karakoç & Alacacı, 2015), so some mathematics topics should still be taught as an abstract concept, that is a concept in which we are difficult to feel or imagine it through our sense (Wilensky, 1991). I still argue that teachers can teach every single mathematics topic using real-world connections primarily for secondary school's students because they learn more abstract concepts that will be harder to find connections with real situations. Furthermore, doing mathematics also can be a sense-making of mathematics even though students do not know when they can use it because everyone uses mathematics differently in his life. A mathematics topic which for someone will not be used in his life, maybe for others, they use it in their daily activities. Thus, the usefulness of mathematics depends on who they are or who they will be.

Furthermore, I do not think all real-life problems are practicable for students because the problems do not relate to their life directly. I did some problems from some websites and a textbook of calculus (SMP, 1973), but not all problems were relevant to a real context and could be solved. I encountered that there was a problem, finding the volume of the box with ignoring its thickness, when some facts were abandoned in order to make students understand the question easily. Moreover, a problem which is relevant to one student's life may not be related to others. Therefore, teachers should check the effectiveness of the problems by asking students first (Burkhardt, 1981), and then they will be able to identify the good problems that can be used in the future. Moreover, calculus is advanced knowledge for most students because they find difficulty to make it concrete so that occasionally it should remain abstract (Wilensky, 1991). Furthermore, teachers need to consider when they give the students real-life problems. They cannot provide them with these problems for every meeting because they also should provide opportunities for students to learn all concepts of calculus, both concrete and abstract. Thus, most teachers assumed the nature of mathematics topics and time restriction may become limitations for connecting it to the real-world (Karakoç & Alacacı, 2015).

However, teachers can motivate students to think inductively in learning calculus. They can involve students to find the first derivative pattern by using the gradient of a straight line and limit concept. They may not give a pattern $f'(x^n) = nx^{n-1}$ directly to students when introducing differentiation, but they should ask students to establish the first derivative pattern by themselves. I also assume that teachers could use a slope of zero ($f'(x)=0$) for figuring out what is the maximum or minimum value of the function quickly. However, teachers have to ask students to check the graph or the second derivative of the function to find the exact category of the x value (maximum, minimum, or inflexion point). Hence, as a mathematics teacher, I should decide an effective teaching strategy carefully to encourage my students understand calculus concepts easily.

Teachers may also adopt examples of the calculus projects and the application of calculus videos explaining the usefulness of calculus in the real life. Personally, I obtained a new perspective by watching some videos showing activities that teachers did like creating a group project related to the application of calculus. However, teachers should consider the time available because doing a project or watching a video will be time-consuming like what De Bock et al. (2003) stated in his study, that is applying real-world connections in teaching mathematics will be time-consuming.

For explaining and showing them the application of mathematics in the real life, teachers can also use more technology (Van den & Drijvers, 2014) in order to make students are able to imagine how they can use it in the real world. I assume that using technology can give sense of calculus for students. I consider using Geometer's Sketchpad (GSP) and Geogebra (https://www.geogebra.org/) while teaching students how to draw a graph of the function and to look closer at whether the function can be differentiated for every point. It is supported by Leong (2013) who found out that students' attitude towards the learning of graph of functions was improved afer using Geometer's Sketchpad Sketchpad. Ndlovu et al. (2011) also found out that teachers can use Geometer's Sketchpad to represent, visualise, and learn derivative and related concepts of introductory calculus. Geogebra is also an effective open-source, free, and easy-to-use software to learn calculus (Hohenwarter et al., 2008). However, Cuban et al. (2001) assumed that using technology in the classroom may become a complex activity. Some teachers may encounter further difficulty in using these softwares because not all of them can operate it and not every school has technological equipment as well as internet connection.

CONCLUSION

Students are likely to obtain many benefits when learning mathematics and then undertand the use of mathematics concept for solving their real-life problems. Calculus is one of mathematics topics may be difficult to be understood by students especially when connecting the calculus to their life. Therefore, teachers have to know how to connect the calculus to students' life. Teachers should know the meaning of connecting calculus to the real life through solving their own problems. However, some factors can be faced by teachers when teaching and learning calculus then connecting it to real-life problems like students' understanding of calculus concepts, the compexity of students' real problems, teachers' abilities, sources, facilities, as well as teaching and learning methods. In Indonesia, teachers will be able to teach calculus then connect it to solve students' real-life problems. They can give more examples of daily problems related directly to students' life. Technlogy can also become an affective way to tech the connections of calculus to real life.

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