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Supporting elementary school children's skill and concept understanding in solving combinatorics problems

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Abstract

The present research is the preliminary part of a large-scale design research aimed at supporting students to use effective strategies and to reach multiplication concept by designing instructional activities in the topic of multiplication principle covering two-dimensional as well as three-dimensional problems. They were conducted in two cycles of which 10 to 11-year-old elementary school children actively participated. Design research then was chosen to reach the set goal. Specifically, in each cycle, hypothetical learning trajectory was set by applying the principle of Realistic Mathematics Education. Reflecting on the activities of both cycles, the findings suggest that by creating hands-on activity, the students are able to make all different possible combinations of each kind of object. Besides that, stemming from pairing one object to the other kind of objects, the students are simplified to use efficient strategy for the next increasingly complex problems. The increasing number problem facilitates students to grasp the use of multiplication in solving the problems. However, the understanding of students in multiplication should be firmly established by the guide of the teacher since they just made inductive reasoning of the solution patterns of the problems

Keywords: *Realistic Mathematics Education; Design Research; Hypothetical Learning Trajectory*

Introduction

As one of the branches of mathematics, discrete mathematics broadly has served other disciplines such as computer science, engineering, statistics, and probability, etc. It causes the problems of discrete mathematics found in many curricula are mostly in the form of applied mathematics and more familiar for both children and adult compared to some other branches of mathematics. It is then interesting to use such topic as the material for problem-solving in mathematics since one can set a problem which is closely related to children's daily life and challenging for them to solve. In realistic mathematics education, the problem can be considered as context as a path aimed to grasp a mathematical concept (Bakker, 2004) .

One of the topics in discrete mathematics which gets major representation in school curriculum is combinatorics (Kavousian, 2008). Such kind of development implied educational studies in that topic also quite evolve. Some of them related to the strategies and the abilities of students use in solving the combinatorial problem (Coenen, Hof, & Verhoef, 2016; English, 2007; Kavousian, 2008) and the importance of combinatorics in solving probability problems (Batanero, Navarro-Pelayo, & Godino, 1997; Quadling, 1974).

In specific to the research of the studies of students' strategies of covering multiplication principle problem, there are some strategies used by elementary school children in solving two-dimensional problems and three-dimensional problems highlighted. In addition, the efficiency of those strategies is also emphasized. The trial and error approach and the odometer pattern were respectively considered as the most inefficient and efficient strategy. The latter strategy which was named since it resembles the odometer in a vehicle is conceptually and closely related to the multiplication concept since if there are m items in each n and there are n items, then there will be m multiplied by n items in total. Meanwhile, in the three-dimensional strategy, the most useful strategy to the concept formation is major-minor. It is so labeled since there is a major item which is less frequently changed and paired with each minor item. These efficient strategies also definitely represent the concept of multiplication as the introductory part of the combinatorial topic which is mostly studied in secondary level.

Considering the potency of students in extending their competencies in problem solving, the novelty of the topic for elementary school children, the rarity of studies and the need of guiding them to comprehend the concept of multiplication principle, hence, the researchers are interested to design a learning of which it formulates a sequence of activities to assist children to apply those efficient strategies and to grasp the multiplication concept using realistic mathematics education. Then, the present research question was posed: how can the designed learning activities support elementary school children to apply effective strategies for solving the problem as well as to reach the understanding of multiplication principle concept?

Solving contextual problem as found in most combinatorial topics is not simple. The studies of Batanero et al. (1997) and Hadar & Hadass (1981) found some errors and obstacles in solving combinatorial problems of students. They mainly pointed out that the problem comprehension of students was quite low causing that they incorrectly chose an appropriate method. In addition, related to the topic of this study, English (1991) also found a non-systematic listing method of children in determining all possible items. To anticipate these constraints during the learning and to obtain the aims which had been set, several key kinds of literature were provided as the base of designing the learning, i.e. the strategies in solving combinatorial problems and Realistic Mathematics Education (RME) The strategies meant were systematically found through several studies. Mashiach Eizenberg & Zaslavsky (2004) issued the lack of students' abilities in assuring their solutions using verification strategies i.e. 1) reworking the solution, 2) adding justification to the solution, 3) verifying by considering the reasonableness of the solution, 4) modifying the problem to be simpler, e.g. changing the numbers of objects without changing the meaning of the problem, and 5) using different method of solution. Moreover, English (1991) suggested that the provision of increasingly difficult problems to children resulted in many kinds of strategies ranging from inefficiently random attempts to more efficient ways. In addition, she concluded that the experience in solving sequence of problems led students skillful in using effective strategies.

The choosing of Realistic Mathematics Education (RME) as the approach in designing learning of this study is identified by its functions which not only offers a pedagogical and didactical philosophy on teaching and learning mathematics but also designing instructional materials for learning (Bakker, 2004). In addition, Freudenthal in Quadling (1974) stated that since one of the characteristics of RME which allows students to invent their own strategies in solving problems and leads students to gain the goal of learnings i.e. understanding mathematics concept, RME-based research fits the research question, e.g., posed in this study. The stage of RME crucially highlighted is that how to support students in reaching mathematical concept understanding stemming from their own strategies using a model by the guide of a teacher (Dickinson & Hough, 2012). Van Den Heuvel-Panhuizen (2003) suggested that the model itself is a representation made by the situation of the problem given in which there is a mathematical concept. In this study, the researchers tried to create the guide by creating the activities of which students use the desired efficient strategies as the model and come up with the multiplication as the mathematics concept. The designed learning activity would also apply the tenets of RME (Bakker, 2004), i.e. using context as the outset, using students' own productions, and promoting the interactivity among students to let them freely discuss what have they made.

Materials and Methods

As in this study, a sequence of activities to support students' understanding and skills was designed, design research then was chosen as the method of the research. Gravemeijer (2004) stated there are three phases of design research: the preparation of the experiment, the classroom experiment, and the retrospective analyses. In the preparation phase, a hypothetical learning trajectory (HLT) was designed which consists of learning goals, teaching and learning activities, and conjecture of student's thinking (Bakker, 2004). HLT guides the design of instructional materials that have to be developed or adapted. In addition, HLT can be elaborated and refined while conducting the experiment.

In general, the data in this study were obtained from the preparation of the experiment and the experiment. They were gathered by doing an interview, observing, and collecting written documentation. The interview and the observations were recorded by using field note and video to collect information e.g. the grade and the mathematics ability of students. Documents which were mainly collected in the experimental phases were student's written works. There were six students divided into two groups who actively participated. They were taken as samples by random purposive sampling technique who are heterogeneous in the term of mathematics ability and gender. In addition the students haven't studied multiplication principle.

The issues of the validity and the reliability in this study mainly refer to the study of Miles & Huberman (1994) and the study of Bakker (2004) of which internal validity, external validity, internal reliability, and external reliability should be noticed. They are all concerned in a qualitative way. Internal validity refers to the data collection quality and the considerable reasoning which can be used to draw a conclusion. Then in this study, it was gained by collecting the different types of data (data triangulation) such as video recording, audio recording, photographs, field notes, and written work of the students. We also conducted different teaching experiments in the first cycle and in the second cycle aimed, one of them, to test the conjectures set in the earlier experiment in the later experiment. External validity or the generalizability is the extent to which one can generalize the findings from the contexts used in this study to other contexts which can be issued by presenting the findings of this study clearly so others can transfer it to their domains. Internal reliability means the extent to which the inference and the argumentation are reasonable. In this study, it was improved by discussing crucial activities with colleagues to minimize the sense of subjectivity and doing a careful collection to the data e.g. coding the audio transcript and making video fragment. External reliability means replicability which has a criterion i.e. trackability of which a researcher should report the success of his research in such a way that a reader can track his activities during research.

Based on the reviewed literature and the thought experiments of the researchers with some colleagues, two activities were set in the HLT both in the first HLT and the second HLT. The stipulated goals for the first activity are students can list and determine the number of all possible two-dimensional pair combinations using odometer pattern approach using hands-on activity of which pairing each kind of snacks to each kind of drinks and students can list and determine the number of possible two-dimensional combinations using multiplication without hands-on activity. The problems for the first goal were set, 1-2 (one snack and two kinds of drinks) and 1-3 and there were four problems for the second goal of which there were two kinds of shirts for each problem and two, three, four, and five kinds of trousers respectively for the first, the second, the third, and the fourth problem. Meanwhile, the goals set for the second activity are students can list and determine the number of all possible three-dimensional pair combinations using major-minor strategy of which the problems were set 1-2-3 (one snack, two different drinks, and three different fruits) and 2-2-3 and students can list and determine the number of possible three-dimensional combinations using multiplication of which the problems were set 1-2-2 (one shirt, two different trousers, and 2 different hats), 2-2-2, 3-2-2, and 2-5-4. The data which have been collected were analyzed and interpreted by some colleagues

Results and Discussion

3.1 Activity 1

As being hypothesized, the students did the first and the second problem by pairing the snack with each of the drinks available. Specifically, both groups held the snack and moved it nearby or the top of each drink alternately, thus they got 2 and 3 as the answers respectively. Meanwhile, for the third problem, they applied odometer strategy by firstly pairing one of the snacks to each of the drinks and doing the same for the other snacks. However, unpredictably, this activity is different to the set hypothesis which predicted that the students who used odometer strategy would simply continue pairing the snacks and the drinks from the activity in the second problem, instead, the students started pairing and writing the combinations from the beginning. In determining the total of the combinations, they saw the total of the combinations resulted from their written works.

Moreover, when working without objects provided, i.e. the context of shirts and trousers, surprisingly, the students mentally answered the total of the combinations first before listing the distinct pairs of a shirt and a trouser. They trivially knew that multiplying the number of shirts and the number of trousers is the method to know the total of the combinations. Furthermore, when being given the last problem, the students knew that the answer was 20. When being asked aimed to guide them how they came up with the number of the combinations, they reflected on the solution patterns from the previous problems. The listing they made was just for assuring that the number of the combinations was the same as the number obtained by multiplication. Their conceptions were more firmly established when they could explain that the number could be obtained using addition since they saw from the strategy they used.

3.2 Activity 2

In the first problem, the second group applied the major-minor strategy. Interestingly, it wrote down the drinks first and completed with the snack and the fruits as the second and the third component respectively of which the last component was the most frequently changed component as shown in Figure 1. On the other hand, the first group made the fruits as the minor component and the drinks as the most frequently changing items.

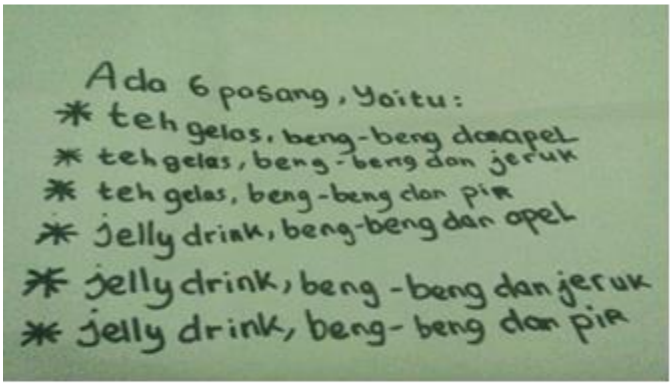

Fig. 1. The work of the second group of the three-dimensional problem

Like in the two-dimensional problem, both groups knew the answer before listing the possible combinations one by one. Based on an interview aimed to know their reasoning, the second group knows that the multiplication was used by reflecting on the results of the previous problems. The consistency of the answer pattern that suits the multiplication of the numbers of each item led the students to use the multiplication to know the number of possible combinations. In addition, after being interviewed regarding the answer, the use of multiplication to get six was also applied by the first group. However, the researchers felt difficulty to explore their ideas since the students simply explained that multiplying the numbers of each item was a simple and a quick way to get the answer as shown in the following interview fragment (coded 007) framed in Figure 2.

In addition, they perceived that the strategy used by the second group was efficient in listing the combinations since it would cause less change when the drinks became the minor component. Moreover, when the number of kinds of snacks was altered becoming two, unlike the previous problem, the first group was not able to directly answer the total number of the possible combinations, instead, it established the combinations one by one first by using major-minor strategy. Specifically, its work was quite like how the second group did the latest problem by forming a pattern of which the students held and wrote the first kind of drinks constantly for the first three combinations while keep maintaining the first kind of snack. These three drinks and snacks than were completed with fruit which was different each other. Later the students continued the pattern with the second kind of drink for the next three combinations whose pattern similar to the previous three combinations. After completing and seeing the whole possible combinations, the students then saw that the total was 12. On the other hand, the second group showed a significant progress by simply multiplying six derived from the possible combinations in the previous problem by two since the second snack also caused the other six combinations. It also asserted that multiplication of each item numbers was used for this kind of problem i.e. 2×3×2. Like the first group, it was capable of using the major-minor strategy for writing every combination for this problem.

Researcher	: why using multiplication? (001)
Reza	: since using the way like this (pointing out the combinations written in the paper work) is harder (002)
Researcher	: but why was it should be the multiplication? (003)
Marvin	: in order to get the result easily, it's faster (004)
Researcher	: but how do you know that it should be multiplication? (005)
Reza	: the problem that was given there were two and two (two snacks and two drinks) becoming four (006)
Marvin	: also there was one snack and three drinks, if it was added becoming four combinations, if being multiplied becoming three combinations, and the answer was three combinations. (007)

Fig. 2. The interview portraiting the argumentation of the students in using multiplication

Conclusion

This study was initiated with the question how elementary school children can be supported to apply effective strategies in solving the problem as well as to reach the understanding of multiplication principle concept. From the learning activities, several conclusions can be suggested. Firstly, reflecting on the first activity in both HLTs, the hands-on activities quite help the students not miss each possible combination. Secondly, the students are quite assisted to use the odometer strategy by the set of activities which starts from one object for one of the items. From such kind of regularity in writing every combination for two-dimensional problem, the students became accustomed and applied the method in using major-minor strategy for three-dimensional problems. Thirdly, the efficient strategies also lead them to connect the solution by listing one by one possible combination to the addition operation and the multiplication operation. The latter operation is identified by the students as the series of problems set in the form of increasingly complex problems which result in an easily identified multiplication-connected-pattern. However, the students' conception of addition and multiplication operation is not connected since all of them did not see the multiplication as a repeated addition as asserted by the teacher of the students in the second cycle based on the interview. It is suggested for further design researchers to put emphasis on the understanding of students toward multiplication principle concept, e.g. providing them a sequence of activities which also introduce multiplication model such as tree diagram.

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References

Bakker, A. (2004). Design research in statistics education: On symbolizing and computer tools. Utrecht University.

Batanero, C., Navarro-Pelayo, V., & Godino, J. D. (1997). Effect of the implicit combinatorial model on combinatorial reasoning in secondary school pupils. *Educational Studies in Mathematics, 32*(2), 181–199.

Coenen, T., Hof, F., & Verhoef, N. (2016). Combinatorial reasoning to solve problems.

Dickinson, P., & Hough, S. (2012). *Using Realistic Mathematics Education in UK classrooms. Centre for Mathematics Education, …*. Retrieved from http://www.hodderarnold.com/SiteImages/eb/eb96bf0d-97df-485b-8e1c-687009630de0.pdf

English, L. D. (1991). Young children's combinatoric strategies. *Educational Studies in Mathematics*, *22*(5), 451–474.

English, L. D. (2007). Children's strategies for solving two-and three-dimensional combinatorial problems. In *Stepping stones for the 21st century: Australasian mathematics education research* (pp. 139–156). Sense Publishers.

Gravemeijer, K. (2004). Local instruction theories as means of support for teachers in reform mathematics education. *Mathematical Thinking and Learning, 6*(2), 105–128.

Hadar, N., & Hadass, R. (1981). The road to solving a combinatorial problem is strewn with pitfalls. *Educational Studies in Mathematics*, *12*(4), 435–443.

Kavousian, S. (2008). Enquiries Into Undergraduate Students â€™ Understanding Of Combinatorial.

Mashiach Eizenberg, M., & Zaslavsky, O. (2004). Students' verification strategies for combinatorial problems. *Mathematical Thinking and Learning*, *6*(1), 15–36.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook*. sage.

Quadling, D. A. (1974). Mathematics as an educational task, by Freudenthal H.. Pp. xii, 680. Dfl 60 paper, Dfl 120 cloth. 1973. SBN 90 277 0322 1, 90 277 0235 7 (D. Reidel). *The Mathematical Gazette*, *58*(404), 145–147.

Van Den Heuvel-Panhuizen, M. (2003). The didactical use of models in realistic mathematics education: An example from a longitudinal trajectory on percentage. *Educational Studies in Mathematics*, *54*(1), 9–35.