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Ethnomathematics: Exploration of Traditional Balinese Flute as Mathematics Learning Resources

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Abstrak

Budaya lokal masyarakat merupakan sumber belajar yang sangat kaya. Pemanfaatan budaya Bali dalam pembelajaran matematika masih terbatas dan perlu dikaji lebih luas dan mendalam. Penelitian ini bertujuan untuk menganalisis unsur etnomatematika yang terkandung dalam suling Bali, menunjukkan potensi penerapan suling Bali dalam pembelajaran matematika, dan menganalisis bagaimana seniman suling Bali memperoleh pengetahuan. Penelitian ini merupakan penelitian kualitatif dengan pendekatan etnografi dengan menggunakan tiga subjek pengrajin suling Bali dan satu subjek seniman suling Bali. Pengumpulan data dilakukan dengan menggunakan tiga metode yaitu studi pustaka, observasi, dan wawancara. Data yang terkumpul kemudian dianalisis secara deskriptif dengan menggunakan analisis domain, analisis taksonomi, analisis komponen, dan analisis tema budaya. Penelitian ini menemukan bahwa unsur etnomatematika yang terkandung dalam suling Bali adalah istilah-istilah lokal yang berkaitan dengan teknik pengukuran dan pengukuran, barisan matematika, dan konsep geometri. Seruling ini berpotensi dimanfaatkan sebagai sumber pembelajaran matematika kontekstual bagi siswa SMP dan SMA. Selain itu, ditemukan bahwa pengetahuan suling Bali yang dimiliki oleh pengrajin diperoleh dari belajar sendiri dan dari warisan nenek moyang mereka.

Kata kunci: *Etnomatematika; seruling Bali; urutan matematika; geometri; pembelajaran matematika.*

Abstract

The local culture of the community is a very rich source of learning. The use of Balinese culture in learning mathematics is still limited and needs to be studied more broadly and in depth. This research aims to analyse the ethnomathematical elements contained in the Balinese flute, to point out the potential application of the Balinese flute in mathematics learning, and to analyse how Balinese flute artists acquire knowledge. This research is qualitative research with an etnographic approach using three subjects of Balinese flute craftsmen and one Balinese flute artist. The research subjects were three Balinese flute craftsmen and a Balinese flute artist. Data were collected by using three methods such as literature study, observation, and interviews. The collected data were then analysed descriptively using domain analysis, taxonomy analysis, componential analysis, and analysis of cultural themes. This research found that the ethnomathematical elements contained in the Balinese flute are local terms related to measurement and measuring techniques, mathematical sequences, and concepts of geometry. This flute has potentials utilization as a source of contextual mathematics learning for student at junior and senior high schools. In addition, it is found that knowledge of Balinese flute possessed by craftsmen was obtained from self-learning and from their ancestors' legacy.

Keywords: Learner-Guided Study, Writing, Analytical Exposition Text.

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1. INTRODUCTION

Mathematics has been a human activity and cannot be separated from human history (Pramitasari et al., 2019; Rohman et al., 2020). Its application is wide both in science and in other branches of science. Mathematics is what mathematicians do with all the deficiencies inherent in human activities or creations (Ernest, 1991; Pramitasari et al., 2019). Mathematics is a science that is integrated with human culture so that it is influenced by the values of human life (Ernest, 1991; Zubaidah, 2019). This shows that philosophically mathematics cannot be separated from human culture and civilization. Different cultures may lead to different mathematical practices. The link between mathematics and culture is called ethnomathematics. It recognizes the different ways of doing mathematics in different sectors of society due to cultural differences. More detail, ethnomathematics is an intersection of ethnographic, ethno modelling, and mathematics studies (Arisetyawan et al., 2014; Suharta et al., 2017). Ethnomathematics can be used as a mode, style, and technique in understanding and explaining the natural and cultural environment. Mathematics is explained in different ways and styles according to the way people understand it in a particular area. There are separate ways for people in understanding and doing mathematics which are of course different from one region to another region (Anggoro, 2016; Rosa et al., 2017). The activity of taking noble cultural values and integrating them in subjects has a strategic position in the implementation of learning, including mathematics learning (Darmayasa, 2018; Mashuri et al., 2019). The integration of ethnomathematics will be able to get students closer to the culture of their nation and indirectly teach students about the positive values contained in their culture (Susilo & Widodo, 2018). This suggests that the integration of ethnomathematics can be a solution in teaching mathematics as well as introducing culture (Maryati & Prahmana, 2018) as mathematics is deeply rooted in culture (Abdullah, 2017; Brandt & Chernoff, 2014). Ethnomathematical ideas will be able to enrich existing mathematical knowledge. Using ethnomathematics, mathematics can be taught modestly by adopting local culture (Huda, 2018). Ethnomathematics is proven to be effective in learning mathematics. According to research conducted by previous the integration of ethnomathematics in learning can improve students' metacognition abilities research (Mutaqin et al., 2021). Some other researchers in Indonesia that explore ethnomathematics can be found in (Pathuddin et al., 2021; Prahmana et al., 2021).

It is in line with previous study that analyze the ethnomathematical elements. One of the studies have purpose to determine the characteristics of ethnomathematical-based teaching materials with a multi-representation approach (Budiarsini, 2020). The results of the research are in the form of ethnomathematical teaching materials with a multi-representation approach of valid, practical, and effective quality (Zayyadi, 2017). Moreover, the exploration of Balinese culture as a source of mathematics learning has been carried out by several researchers. For example, traditional Balinese architecture to produce learning resources for geometry related to shapes and transformations (Suharta et al., 2017). Then, Balinese calendar system to produce mathematics learning resources for learning numbers (Septine et al., 2019; Wijayanti, 2020) explores Balinese culture for learning mathematics related to logic and geometric reasoning. Meanwhile, the use of Balinese context in mathematics learning has a positive impact on student understanding and student learning motivation (Suryawan et al., 2021). Hence, elaboration of Balinese ethnomathematics in order to find another source for learning mathematics is very challenging. The knowledge of Balinese flute artists is related to the mindset of the artists regarding flute making and playing technique and their relationship to the ethnomathematics contained therein. This research aims to analyse the ethnomathematical elements contained in Balinese flute, to explore the ways in which Balinese flute artists acquire knowledge about producing and playing techniques; and to point out the application of ethnomathematical concept of Balinese flute in mathematics teaching and learning.

2. METHODS

In this study, we used qualitative methods with an ethnographic approach (Sugiyono, 2016). Ethnomathematics seeks to understand the roles of mathematics in different groups and nations (Burkhardt, 2008). Ethnography approach is a qualitative approach that describes culture (Abdullah, 2017; Evans & Cleghorn, 2022) and provides answers to the question of an individual group about its culture (Sulasteri et al., 2020). The object of this research is the ethnomathematical elements contained in Balinese flute. The research subjects were three Balinese flute craftsmen and a Balinese flute artist. The design of this research uses the framework of ethnomathematics study using four general questions related to Balinese flute (Prahmana & D'Ambrosio, 2020; Utami et al., 2019). Data were collected mainly by using interviews, observation and documentation in natural settings (natural conditions). The collected data were then analyzed using domain analysis, taxonomy analysis, componential analysis, and analysis of cultural themes, referring. The domain analysis including two aspects which are language and knowledge system, as depicted in Table 1.

Table 1. Domain Analysis

No	Domain	Semantic relations	Domain details
1	Language	Unique terms	Term used in the process of manufacturing Balinese flutes to make the process easier.
2	Knowledge system	Calculation procedure	Non-formal calculation procedure used in the process of manufacturing Balinese flutes. Calculation procedures and measurements that are used to make flute with a good and harmonious note.

Base on Table 1 the domains obtained in the previous stage are described in detail with focused observation in the taxonomic analysis, each of which is contrasted in the component analysis to find "red threads" that link one to another in the last analysis.

3. RESULTS AND DISCUSSION

Result

Balinese flute is one of the Balinese wind instruments made from bamboo. There are four types of Balinese flutes, namely small-sized, medium-sized, large-sized, and gambuh flutes, each of which has a different length. The distance between the finger holes on each type of flute is also different. This difference produces different basic notes and uses. Generally, The Balinese flute has one embouchure hole which is called a *song manis* and six holes that are played by closing and opening them alternately using fingers, which are called *song nyawan*. In the process of making Balinese flutes, a standard called sikut is used. Sikut is process to point the position the third hole from the bottom end of the flute, to determine the size of the hole diameters, and to set up the spacing of the holes on the flute. A small Balinese flute, which is usually used to accompany gong kebyar or semarpegulingan—two types of instrument of Balinesse orchestra. In playing the flute, there is a rule for closing and opening the finger holes so as to produce certain note patterns called kupakan or tatekep. When it is associated with the semarpegulingan gamelan, there are seven main notes, namely nding, ndong, ndeng, ndeung, ndung, ndang, and ndaing. When they are linked to diatonic notes, they will match the notes mi, fa, sol, la, si, do, and re (Nita & Ramanathan, 2019). There are several types of kupakan that are named based on the lowest note of which the kupakan can reach. For example, kupakan ndeng means that the lowest note that can be reached by kupakan is ndeng note. When it is examined from the pattern of playing, the bottom hole and the fourth hole from the bottom are always closed. Although they are opened occasionally, they will usually be opened simultaneously with the opening the hole above them. In that sense, if they are wanted to be closed, then while using kupakan ndeng, the bottom and fourth holes from the bottom are always closed. However, if the bottom and the fourth hole from the bottom are open, then the first hole must be opened simultaneously with the second hole and the fourth hole must be opened simultaneously with the fifth hole.

In the language domain, we obtain various unique terms called jargon in the making of Balinese flutes that is related to mathematical calculation which are *ping*, *dum*, *mebed*, and *ngajug*. The term *ping* means “to multiply” or “being multiple”, *dum* means “to divide”, *mebed* means “to encircle”, and *ngajug* means “the process of determining the size and position of the finger hole”. In addition, there are various numerical terms and numerals in regional languages such as *besik* (one), *dua* (two), *telu* (three), and so on. These numerical terms and numerals are part of Balinese number system. In the knowledge system, we found term *sikut* from literature study. *Sikut* means the determination of hole spacing and hole diameter on the Balinese flute. Each flute craftsman has his own characteristics and identity which can be seen from his work. Like a signature affixed to every painting, a flute craftsman also has special marks on his work that can be considered as "signatures". These signs can be in the form of differences in *sikut*, hole shapes, *suwer* shapes, decorations and others. There is a "red threads" that links one domain to another. In making Balinese flutes, one must pay attention to the *sikut* and knowledge of the basics of measurement and calculation. In making measurements and calculations, jargon or related unique terms are used. All of them must be considered in order to make a good-quality flute. In the process of making Balinese flutes, the flute craftsman determines the distance of the hole and the diameter of the hole by making direct measurements using only *a katik* — a stick made by bamboo blade. The craftsman first measures the midpoint of the length of the flute (bamboo). The trick is to measure using a bamboo blade as in Figure 2. After obtaining the midpoint, the craftsmen then measured the diameter of the flute as shown in Figure 3.

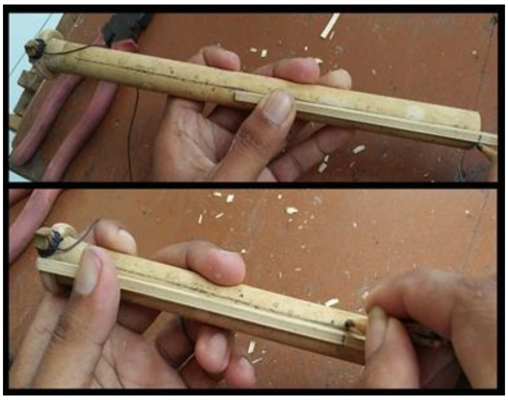

Figure 2. Determining the midpoint

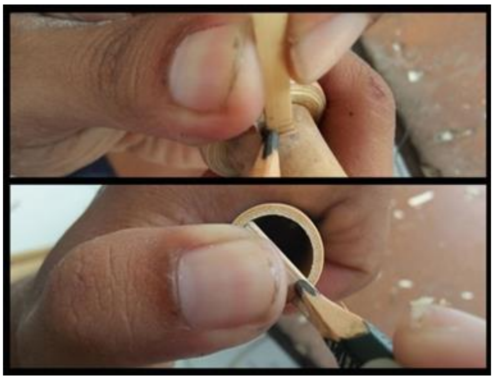

Figure 3. Determining the flute diameter

Then, at the center of the flute, the sketch of the holes is made. The distance between the finger holes of the flute is one time the diameter of the flute, also known as *angajug*. We asked question about the initial process of making Balinesse flute is show in Table 2.

Table 2. Interview of initial process of making Balinese flute

Interview	Translation
"*Simalu alih as ne. Kene carane, nah berarti kan ne as ne, jani bolongne menyesuaikan masi, yen mone tiinge da baanga bolong bes gede, nah artine alih pangusne, paling sing mone baang. Nah yen suba jani megambar bolongne, alih jani angajug. Ngajug dini*."	First of all, locate the midpoint like this (He pointed the middle of the bamboo while showing how to measure the center point of the flute). This point is for the fifth hole from the bottom. The hole is proportional to the length of the flute (neither too big nor too small). After making the hole, continue with *angajug*. *Angajug* is here (showing how to measure).

Base on Table 2, the distance between the finger holes is not the same for each type of Balinese flute. In the large-sized flute and the gambuh flute, the distance between the third and fourth finger holes is equal to twice the diameter of the flute plus the diameter of one finger hole, while in small-sized and medium-sized flutes, the distance between the third and fourth finger holes is the same as the diameter of the flute. The next process is the measurement of the distance between the bottom hole and the bottom end of the flute. The process of measuring the distance between the lowest hole and the lower end of the flute can be seen in Figure 4.

Figure 4. Measuring distance of finger hole and bottom of flute

Base on Figure 4 the distance of the bottom hole of a small-sized flute to the bottom of the flute is twice the diameter of the flute plus twice the diameter of the finger hole. The distance of the bottom hole of a medium-sized flute to the bottom end of the flute is twice the diameter of the flute plus three times the diameter of the finger hole. The distance of the bottom hole of a large-sized flute from the bottom end of the flute is three times the diameter of the flute plus three times the diameter of the finger hole. As for the *gambuh* flute, it is five times the diameter of the flute plus seven times the diameter of the finger hole.In making Balinese flute, the distance between holes is closely related to the concept of sequences and series in mathematics as described in detail below. These concepts found in Balinese flute are different from the concepts in formal mathematics (Nita & Ramanathan, 2019). *Arithmetic Series in a Small-sized Flute*. By assuming that the distance from the center of the bottom hole to the bottom of the flute is *a*, the distance between the two successive center points of the hole is *b* and the length of the flute is *l*. As show in Figure 5.

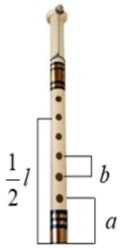

Figure 5. Modeling the *Sikut* of small-sized flute

Base on Figure 5, since distance between two successive finger holes is constant, the distance of the *n*th hole canter point from the lower end of the flute can be determined by finding the *n*th term of the arithmetic sequence.

$$U_n = a + (n-1)b \text{ where } n = 1, 2, \ldots, 6 \quad (1)$$

Supposing that the diameter of the flute is d_1, the radius of the flute is r_1, the diameter of flute's hole is d_2 and the radius of the finger hole is r_2 then, since the distance between the lowest hole and the tip of the flute is twice the diameter of the flute plus twice the diameter of the finger hole, the value of *a* is obtained as follows.

$$a = 2d_1 + 5r_2 \quad (2)$$

Then, because the distance of each hole is equal to the diameter of the flute the value of *b* is obtained as follows.

$$b = d_1 + 2r_2 \quad (3)$$

The spacing of *n*th hole can be determined by substituting eq (1), eq (2), eq (3) to eq (1) so that it is obtained

$$U_n = d_1(n+1) + r_2(2n+3) \quad (4)$$

Next, the center point of the fifth hole is the midpoint of the length of the flute, or it can be said that the distance of the center point of the fifth hole from the bottom end of the flute is $\frac{1}{2}l$ with l is the length of the flute. On the other hand, the distance to the center of the fifth hole is U_5 of the arithmetic sequence on eq (4). If it is substituted, the correct size of the radius of the finger hole is

$$r_2 = \frac{1}{26}l - \frac{6}{13}d_1. \quad (5)$$

Thus the radius of the finger hole must be greater than 0 and less than the radius of the bamboo. Hence, the relation between the length of the flute and the radius of the bamboo is

$$24r_1 < l < 50r_1. \quad (6)$$

Arithmetic Series in a Medium-sized Flute. It is almost the same as making a small-sized flute. First of all, a bamboo with finger holes that has already been tuned for its basic note is then measured in length and then the *sikut* are determined. The only difference lies in the distance between the bottom hole and the bottom of the flute, which is twice the diameter of the flute plus three times the diameter of the finger hole. Using the same arithmetic sequence calculation process as desribed in the previous section, the *n*th hole distance is obtained as.

$$U_n = d_1(n+1) + r_2(2n+5). \quad (7)$$

Next, the center point of the fifth hole is the midpoint of the length of the flute, or it can be said that the distance of the center point of the fifth hole from the bottom end of the flute is $\frac{1}{2}l = U_5$. Thus, if it is substituted, the correct size of the radius of the flute hole is

$$r_2 = \frac{1}{30}l - \frac{2}{5}d_1. \quad (8)$$

So the length of the flute must range between

$$24r_1 < l_1 < 54r_1 . (9)$$

Arithmetic Series in a Large Sized Flute. For a large Balinese flute, the length of the flute is *l* and the diameter of the flute is d_1 as illustrated in Figure 6.

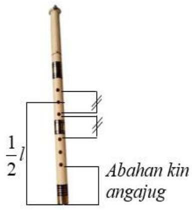

Figure 6. *Sikut* of large-sized flutes

Base on Figure 6, between the third and fourth holes, it is as if there is one more hole. Determining six holes on a large flute is actually the same as making seven holes, each of which is one flute diameter, but the fourth hole is not perforated. So, the holes that are perforated are only the first, second, third, fifth, sixth, and seventh holes. Further, for simplicity of calculations, it is assumed that the large flute has seven holes but the fourth hole is not perforated. Furthermore, the calculation is carried out using the arithmetic sequence rule as before. Since the distance from the first hole to the bottom end of the flute is three times the diameter of the finger hole and the distance between the finger holes is equal to the diameter of the flute, then the *n*th hole distance is obtained as

$$U_n = d_1(\mathrm{n}+2) + r_2(2n+5). (10)$$

Next, the center point of the fifth hole is the midpoint of the length of the flute, or it can be said that the distance of the center point of the fifth hole from the bottom end of the flute is $\frac{1}{2}l = U_6$. Thus, if it is substituted, the correct size of the radius of the finger hole is

$$r_2 = \frac{1}{34}l - \frac{8}{17}d_1. \ (11)$$

Then, as explained earlier, that the radius of the finger hole must be more than zero and must be less than the radius of the flute. So the length of the flute must range between

$$32r_1 < l_1 < 66. \ (12)$$

Arithmetic Series in a Gambuh Flute. Making a *gambuh* flute is the same as making a large flute. The difference between the *gambuh* flute and the large flute is only in the distance between the bottom hole and the lower end of the flute. In the *gambuh* flute, the distance from the first hole to the lower end of the flute is five times the diameter of the flute plus seven times the diameter of the finger hole. In the same way, the distance of the nth hole will be obtained as

$$U_n = d_1(\mathrm{n}+4) + r_2(2n+13). (13)$$

The center point of the fifth hole is the midpoint of the length of the flute, or it can be said that the distance of the center point of the fifth hole from the bottom end of the flute is $\frac{1}{2}l = U_6$. Thus, if it is substituted, the correct size of the radius of the finger hole is

$$r_2 = \frac{1}{50}l - \frac{2}{5}d_1. (14)$$

Then, as explained earlier, that the radius of the finger hole must be more than zero and must be less than the radius of the flute. So the length of the flute must range between

$$40r_1 < l_1 < 90r_1. (15)$$

Geometric Series in The Chromatic Scale Frequency and the Length of Balinese Flute. Practicaly, Balinese flutes is commonly used to accompany *arja* dances dan *geguntangan* (an art that combines *tembang* (song) and *gambelan* (music)). When they accompanying *arja* dances or *tembang* in *geguntangan*, the flute player will follow the strains of the *tembang* sung by *arja* dancers and the singer. Therefore, the note of the flute used must match the tone of the song sung by the *arja* dancer or singer. Meanwhile, *arja* dancers or singer will usually use changing basic notes when they sing *tembang*. This is because the *tembang* note is sung according to the character of the song. Consecuently, it is not enough for flute players to use only one flute. Therefore, several flutes are needed to be able to cover all the notes that a singer might sing. Usually a flute player will provide about 12 flutes, or even more, so that all the tones are represented. A set of flutes consisting of 14 flutes used to accompany the *arja* dance is known as *suling pangarjaan* as shown in Figure 7.

Figure 7. A set of a *Pangarjaan* Flute

Traditionally, there is no specific term to name each of the basic notes of Balinese flute. However, in fact, the basic notes used in a set of *suling pangarjaan* fall into a chromatic scale. If the basic note of the flute is measured and then matched with a modern scale, the flute notes will correspond to the chromatic scale as shown in Table 3.

Table 3. Flute Tone Frequency on *Suling Pangarjan*

No	Flute length (cm)	Flute diameter (cm)	Basic tone	Basic tone frequency (Hz)
1	19	1.2	$G^{\#}$	837.4
2	20.7	1.3	G	776.4
3	20.9	1.3	$F^{\#}$	749.7
4	22.9	1.3	F	698.2
5	24.2	1.4	E	655.1
6	25.9	1.4	$D^{\#}$	618.4
7	27.25	1.4	D	583.4
8	28.9	1.6	$C^{\#}$	550.3
9	29.05	1.9	C	523.2
10	32.15	1.7	B	494.3
11	32.9	1.7	$A^{\#}$	467.5
12	35.3	1.8	A	443.5
13	36.65	2.1	$G^{\#}$	423.8
14	39.6	2.05	G	388.9

The frequency of a note in the upper octave is twice the frequency of the note in the lower octave. The number of notes that make up the chromatic scale is 12 notes. There are C, $C^{\#}$, D, $D^{\#}$, E, F, $F^{\#}$, G, $G^{\#}$, A, $A^{\#}$, B, and continuing to the note C, which is an octave above the original C note (Worland, 2013). Since every two adjacent notes have the same pitch interval, to determine the frequency of the other notes, it is the same as inserting 11 terms in the geometric sequence to a ratio of 2. So that the ratio between the note frequencies is $r = \sqrt[12]{2} = 2^{\frac{1}{12}}$. From the relation of the note frequencies, it can be formulated that the *n*th note frequency will follow the *n*th term rule of the geometric sequence with the formula for the *n*th term is $U_n = f \times 2^{\frac{1}{12}(n-1)}$.

When the basic note is tuned, the length of the flute is cut in small increments until it reaches the desired note. The basic note produced by a flute is closely influenced by the length of the flute. The shorter the flute is cut, the higher the basic note will be. If the length of the flute is related to the frequency of the chromatic scale, a pattern will be obtained to determine the appropriate length of the flute to produce a note with a certain frequency. According to the principle of the open pipe organ (Worland, 2013), the flute is required to produce notes with wavelength λ that is half of its wavelength (Mutaqin et al., 2021; Sulasteri et al., 2020). To produce an upper octave note, the wavelength must be shortened to half of the original length. Thus, the length of the flute needed to produce the basic note in the upper octave is half of the length of the flute with the basic note one octave lower. Since there are 12 notes in the chromatic scale sequence, the length of the flute to produce other notes can be calculated by inserting 11 terms in the geometric sequence with half of the ratios. So that the ratio between the tone frequencies of $r = \sqrt[12]{\frac{1}{2}} = \left(\frac{1}{2}\right)^{\frac{1}{12}}$ is obtained. To determine the length of the *n*th flute, the formula of $U_n = l \times \left(\frac{1}{2}\right)^{\frac{(n-1)}{12}}$ is used.

Geometric Concepts. The Balinese flute contains geometric concepts, both plane geometry and solid geometry. The concept of plane geometry found in the Balinese flute is a circle which can then be further developed into the concept of its radius and diameter. The concept of concentric circles also exists in the Balinese flute which can be seen from its cross section as shown in Figure 8.

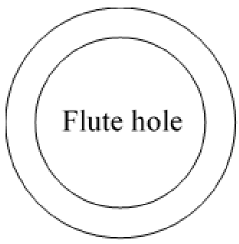

Figure 8. Cross section of Balinese flute

Meanwhile, the solid geometric concept contained in the Balinese flute is a cylinder that can be seen directly from its physical form as sketched in Figure 9.

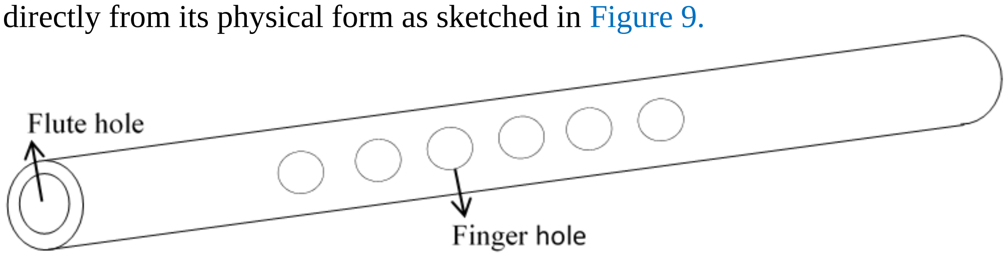

Figure 9. Skecth of Balinese flute

The concepts of height of cylinder and volume of cylinder can also be elaborated on the physical form of Balinese flue. *The Process of Acquiring Knowledge*. Based on interviews, knowledge about Balinese flutes was mostly obtained by learning from craftsmen or senior artists, reading literature about Balinese flutes such as *lontar*, and experimenting and determining their own creation. As expressed by the second flute craftsman, Mr. I Nyoman Sadra, he started his learning to make flutes due to his interest in this musical instrument since childhood. Due to his curiosity about the flute, he finally tried to make Balinese flutes. Because his work was not good at that time due to his ignorance of the *sikut* of the flute, he finally asked an older flute artist named Mr. I Wayan Gabluh. Due to his perseverance, he finally mastered the procedures for making Balinese flutes. When Mr. Sadra was asked to question about his initial interested and experienced in making Balinese flute, he answered as show in Table 4.

Table 4. Interview interested and experienced in making Balinese flute

Interview	Translation
"*Malune nak demen. Nah pidan bapan pak e dini kan medagang acung, mulih uli pasar seni meli pak suling misi gambaran naga keto, aba mulih, ked jumah to lantas tempa. Lantas sikut ne sing luung ban ngae, metakon lantas kemu sik Pak Gabluhe, kengken ane beneh sikute.*"	At first I was very interested. My father has been a street vendor when I was young. He bought a flute with dragon painting. I duplictated the flute. But the result was not good. Its *sikut* was not good. I then asked Mr. Gabluh [about] how to make the right *sikut*.

Discussion

The ethnomathematical elements found in Balinese flutes are mathematical concepts related to number, plane geometry, and solid geometry. Arithmetics and geometric series are found in distance of the flute holes. Plane and solid geometri are found in the form of the flute. It is also found the methods of measurement and calculation. These methods related to the size of hand and finger length. These two methods are used uniquely by Balinese flute craftsmen in producing their flute so that they become the characteristics of the craftsmen. Building knowledge about making Balinese flute is also an interesting result. Balinese flute craftsmen and artists gain knowledge from the knowledge of their predecessors, learn from their own experiences and learn from those who are more skilled. The distance of the center of the finger holes relative to the lower end of the flute forms arithmetic sequence while the chromatic scale frequencies and length of Balinese flute form a geometric sequence, both of which can be used as mathematics learning resources for grade 8 or grade 11. Learning activities that emphasize the application of Balinese Flute ethnomathematics for students in grade 8 are determining the nth arithmetic sequence. They can be done by taking measurements on the Balinese flute and then analyzing the data obtained by students. For upper-level students, such as grade 11 students, learning activities can be carried out by measuring and analyzing the Balinese flute theoretically so that a geometric sequence pattern is found at the note frequency of the flute. Meanwhile, geometry topics such as circle, radius of circle, diameter of circle, cylinder, height of cylinder, and volume of cylinder are topics of mathematics for student in junior and senior high school level.

It is in line with previous study that analyses Balinese culture related to Balinese calendar could be a potential source of learning mathematics in multiple, common multiple, and the least common multiple (meeting) of numbers (Suharta et al., 2017). Here we complete the source with arithmetic's and geometrics series which are more advanced concepts in numbers. It is found that Balinese culture related to traditional house carvings contain a concept of measurement such as *lengkat*, *nyari*, and *rai*, similarity, shift and reflection which are important concept in geometry. Whereas the other study found in Balinese dance *Legong* and its clothing contain geometry concept related to plane figure, circumference or surface area of polygon which are also important concept in plane geometry (Septine et al., 2019). Here we complete with the concept of concentric circle, height and volume of cylinder, and surface area as well. Our findings are not only related to plane figure but also related to solid geometry figure. The implication of this research is to reintroduce students about local culture and become an alternative in mathematics learning media. Then the limitation in this study only lies in the involvement of the wider subject. So it is hoped that other researchers in the future will analyse it more deeply.

4. CONCLUSION

Ethnomathematics of Balinese flute can be integrated in mathematics learning, especially in arithmetic sequence, geometric sequence, and geometry topics. Thus, Balinese flute has potensial utilization as a source of contextual mathematics learning in schools for student at junior and senior high schools. Moreover, these concepts can be used as integrated materials in learning with a Science, Technology, Engineering and Mathematics (STEM) framework. In applying the STEM framework in learning, at least two concept from STEM element must be implemented.

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